Development and Implementation of a Knowledge Aided STAP Algorithm

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Outline

- **Background**
- **Knowledge-Aided Data Pre-Whitening**
- **Example Applications**
  - Distributed clutter
  - Discrete clutter
  - Combined discrete and distributed clutter
- **Real-Time Implementation Analysis**
  - Implementation approach
  - Real-time demonstration status
- **Future Plans and Summary**

Background

- **Clutter cancellation based on *a priori* knowledge alone will often not result in adequate performance**
- **Focus will be on techniques that combine or "blend" adaptive and deterministic filtering**
- **The desired approach is to cancel the known interference component (data pre-conditioning) → follow up with adaptive processing → will require fewer DoFs**
- **Improves performance in heterogeneous clutter environments**
  - Localized training to better "match" the clutter notch width
  - Overcome limited sample support caused by editing techniques
- **Looking for techniques with these properties:**
  - Combines deterministic (knowledge-aided) and adaptive filtering in a way that reduces the overall number of training samples
  - Can be implemented/integrated into existing data domain STAP processing architectures

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Interference Modeling

- **Assume the clutter signal plus thermal noise model**

$$\mathbf{x} = \mathbf{x}_c \circ \mathbf{t} + \mathbf{n}$$

$\circ$ **- Hadamard element-wise product**

- **The modulation is typically small**

$$\mathbf{t} = \mathbf{1} + \mathbf{d}$$

$\mathbf{d}$ **is zero-mean, variance << 1**

- **Clutter signal with small modulation**

$$\mathbf{x} = \mathbf{x}_c + \mathbf{x}_c \circ \mathbf{d} + \mathbf{n}$$

- **Clutter correlation matrix**

$$E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{R}_{\mathbf{xx}} = E\{\mathbf{x}_c\mathbf{x}_c^H\} + E\{\mathbf{x}_c\mathbf{x}_c^H\} \circ E\{\mathbf{d}\mathbf{d}^H\} + \sigma^2\mathbf{I}$$

$$= \underbrace{\mathbf{R}_c}_{\text{“known” component}} + \underbrace{\mathbf{R}_c \circ \mathbf{T} + \sigma^2\mathbf{I}}_{\text{unknown component}}$$

“known” component

unknown component

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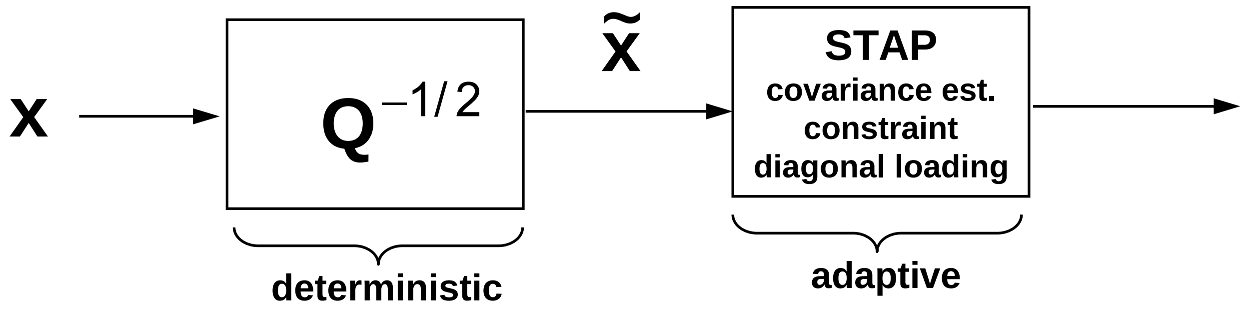
KA Signal Processing Approach



- **The desired approach is to cancel the known interference component (data pre-conditioning)**
  - Full ground clutter model
  - Discretes
  - Jammers/RFI
- **Follow up with adaptive processing → will require fewer DoFs**
- **This two-stage form does not readily “fit” existing data domain STAP implementations**

Knowledge-Aided Quadratic Constraints

Reduced-DoF STAP

- **We can incorporate the reduced-DoF covariance model as a quadratic constraint**

$$\min_{\mathbf{w}_m} \quad E\{|\mathbf{w}_m^H \mathbf{x}_m|^2\} \quad s.t. \quad \begin{cases} \mathbf{w}_m^H \mathbf{v}_m = 1 \\ \mathbf{w}_m^H \mathbf{R}_{c,m} \mathbf{w}_m \le \delta_{d,m} \\ \mathbf{w}_m^H \mathbf{w}_m \le \delta_{L,m} \end{cases}$$

want weights to be "orthogonal" to the reduced-DoF *a priori* clutter model

this is the KA part

- **Gives:**

$$\mathbf{w}_m = \frac{(\mathbf{R}_m + \beta_{d,m}\mathbf{R}_{c,m} + \beta_{L,m}\mathbf{I})^{-1}\mathbf{v}_m}{\mathbf{v}_m^H(\mathbf{R}_m + \beta_{d,m}\mathbf{R}_{c,m} + \beta_{L,m}\mathbf{I})^{-1}\mathbf{v}_m} = \frac{(\mathbf{R}_m + \mathbf{Q}_m)^{-1}\mathbf{v}_m}{\mathbf{v}_m^H(\mathbf{R}_m + \mathbf{Q}_m)^{-1}\mathbf{v}_m}$$

"colored loading"

- **Same form as full-DoF case**
- **Can be readily implemented in the data domain**
- **Can also be shown to be a prefilter on the reduced DoF data (next slide)**

Pre-Filter Interpretation

- **Colored loading beamformer can be expressed as:**

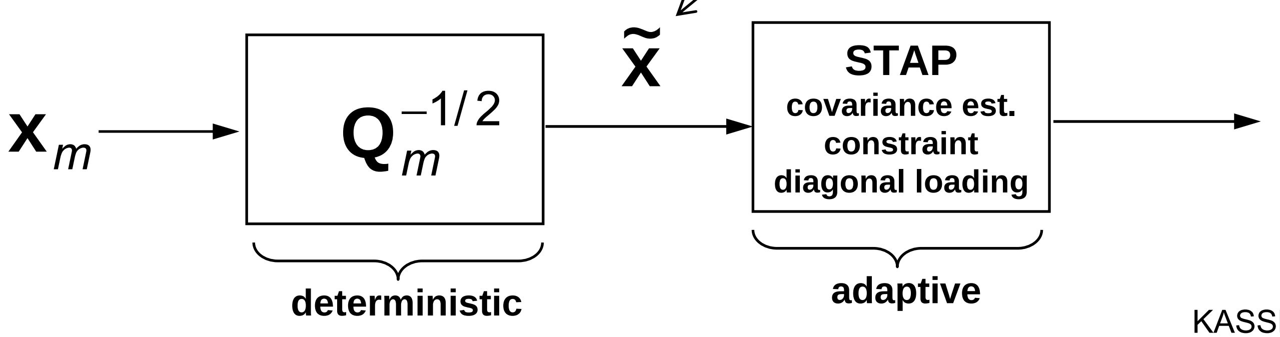
$$\mathbf{w} = \frac{\mathbf{Q}_m^{-1/2}(\mathbf{Q}_m^{-1/2}\mathbf{R}_m\mathbf{Q}_m^{-1/2} + \mathbf{I})^{-1}\mathbf{Q}_m^{-1/2}\mathbf{v}_m}{\mathbf{v}_m^H\mathbf{Q}_m^{-1/2}(\mathbf{Q}_m^{-1/2}\mathbf{R}_m\mathbf{Q}_m^{-1/2} + \mathbf{I})^{-1}\mathbf{Q}_m^{-1/2}\mathbf{v}_m}$$

- **This filtering solution is equivalent to *deterministic* pre-filtering followed by *adaptive* processing (i.e., 2 stages)**

$$\tilde{\mathbf{x}} = \mathbf{Q}_m^{-1/2}\mathbf{x}_m \quad \tilde{\mathbf{v}} = \mathbf{Q}_m^{1/2}\mathbf{v}_m$$

$$\tilde{\mathbf{w}} = \frac{(\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} + \mathbf{I})^{-1}\tilde{\mathbf{v}}}{\tilde{\mathbf{v}}^H(\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} + \mathbf{I})^{-1}\tilde{\mathbf{v}}}$$

it will generally be easier to estimate the interference covariance of the pre-filtered data than the original data because it is likely to have a lower *effective* rank



Constraint Satisfaction

- **The two loading levels are determined by assuring satisfaction of the two soft constraints**
- **Leads to two coupled non-linear inequality relations for the two real scalar loading levels embedded in Q**

$$\mathbf{v}^H(\mathbf{R_{xx}}+\mathbf{Q})^{-1}\mathbf{R}_c(\mathbf{R_{xx}}+\mathbf{Q})^{-1}\mathbf{v} \le \delta_d\left(\mathbf{v}^H(\mathbf{R_{xx}}+\mathbf{Q})^{-1}\mathbf{v}\right)^2 \quad (1)$$

$$\mathbf{v}^H(\mathbf{R_{xx}}+\mathbf{Q})^{-2}\mathbf{v} \le \delta_L\left(\mathbf{v}^H(\mathbf{R_{xx}}+\mathbf{Q})^{-1}\mathbf{v}\right)^2 \quad (2)$$

- **No closed form solution, must be solved iteratively**
- **In the white noise gain relation, $\delta_L > 0$ to obtain solution**
- **In the clutter orthogonality relation, reducing δ_d requires that the colored loading level β_d be increased**
- **In the limit of $\delta_d \rightarrow 0$ (true orthogonality of weights to the clutter model), $\beta_d \rightarrow \infty$**
- **This can be demonstrated directly by comparing the quadratic constraint weight solution with a multiple linear constraint weight solution that enforces orthogonality explicitly**

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Distributed Clutter Colored Loading

- **The ground clutter covariance model is given as**

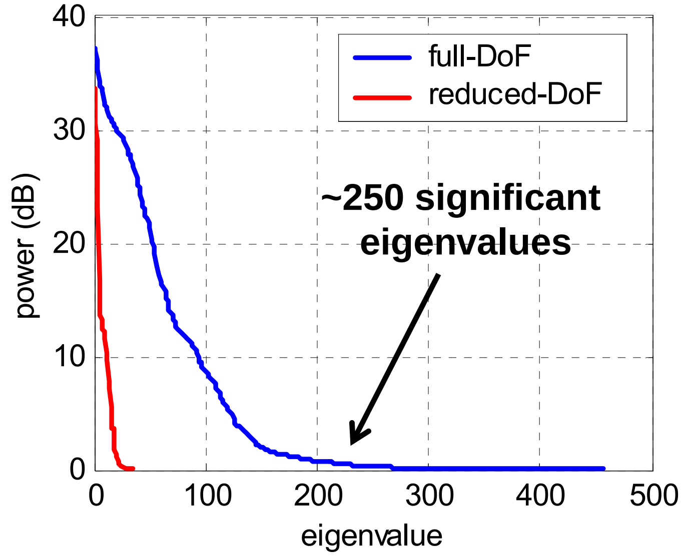
$$\mathbf{R}_c = \sum_{p=1}^{N_c} \left|\alpha_p\right|^2 \mathbf{v}(\theta_p, f_p)\mathbf{v}(\theta_p, f_p)^H \quad \mathbf{R}_{c,m} = \mathbf{H}_m^H \mathbf{R}_c \mathbf{H}_m$$

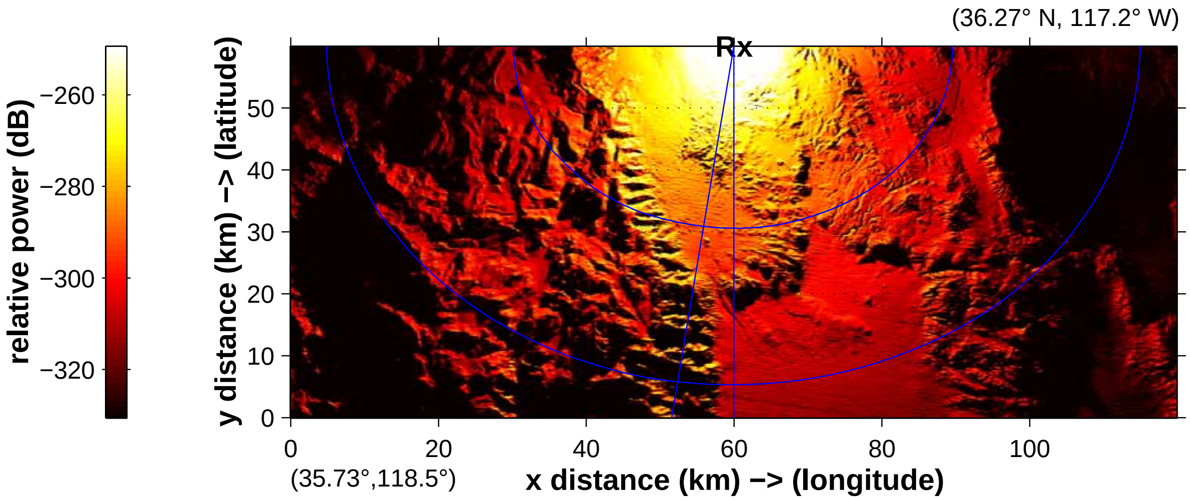
- **Where α_p may include assumed Tx and Rx subarray patterns and the ground clutter power based on a site-specific terrain model (e.g. DTED)**
- **Unknown quantities in the covariance model analyzed:**
  - Angle independent calibration errors (gain and phase)
  - Angle dependent calibration errors
  - ICM
  - Terrain (α_p = 1)
  - transmit pattern
- **The knowledge-aided STAP weights are computed as**

$$\mathbf{w}_m = \kappa(\mathbf{R}_{s,m} + \beta_{L,m}\mathbf{I} + \beta_{d,m}\mathbf{R}_{c,m})^{-1}\mathbf{v}_m$$

KASSPER Data Set 2

Parmeter	Value
RF frequency	10000 MHz
Bandwidth	10 MHz
PRF	2000 Hz
Peak Power	10 kW
Duty factor	10%
Noise figure	5 dB
System losses	7 dB
Platform speed	150 m/s
Platform altitude	7 km ASL
Receive aperture	1.425 m x 0.285 m
Number of subarrays	12
crab	~3 degrees
Number of Pulses	38





- **Site-specific data set generated under KASSPER program**
- **Heterogeneous clutter, ground vehicles, ICM, calibration errors**
- **Eigenvalues shown for range bin 100**

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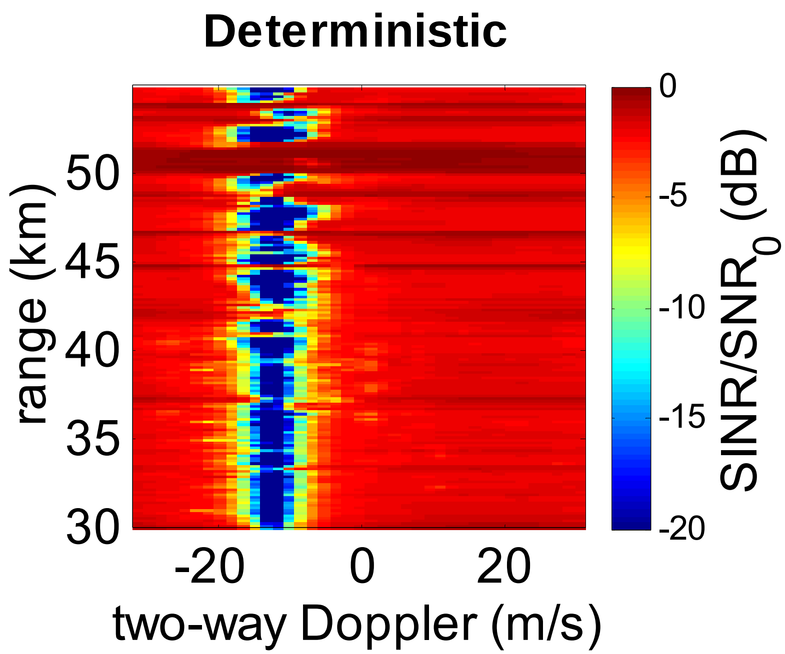
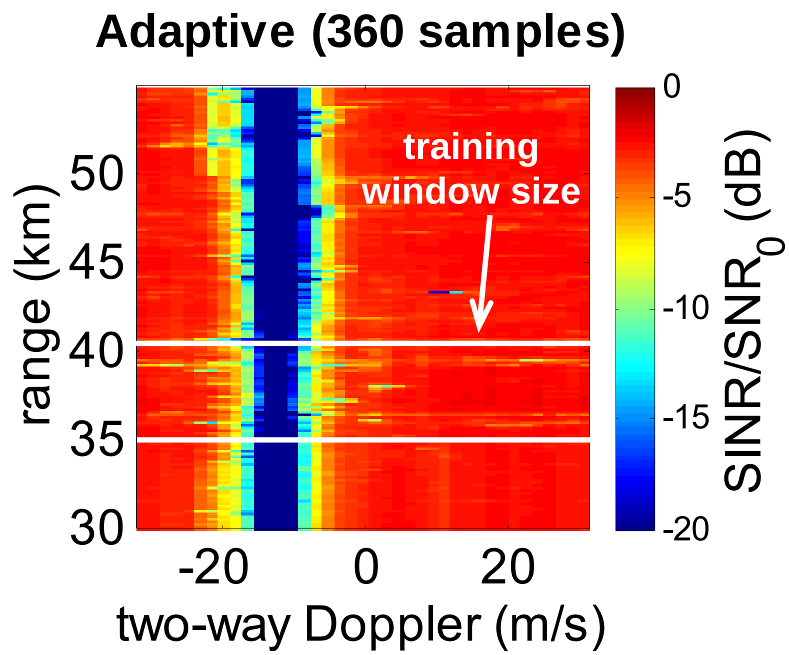
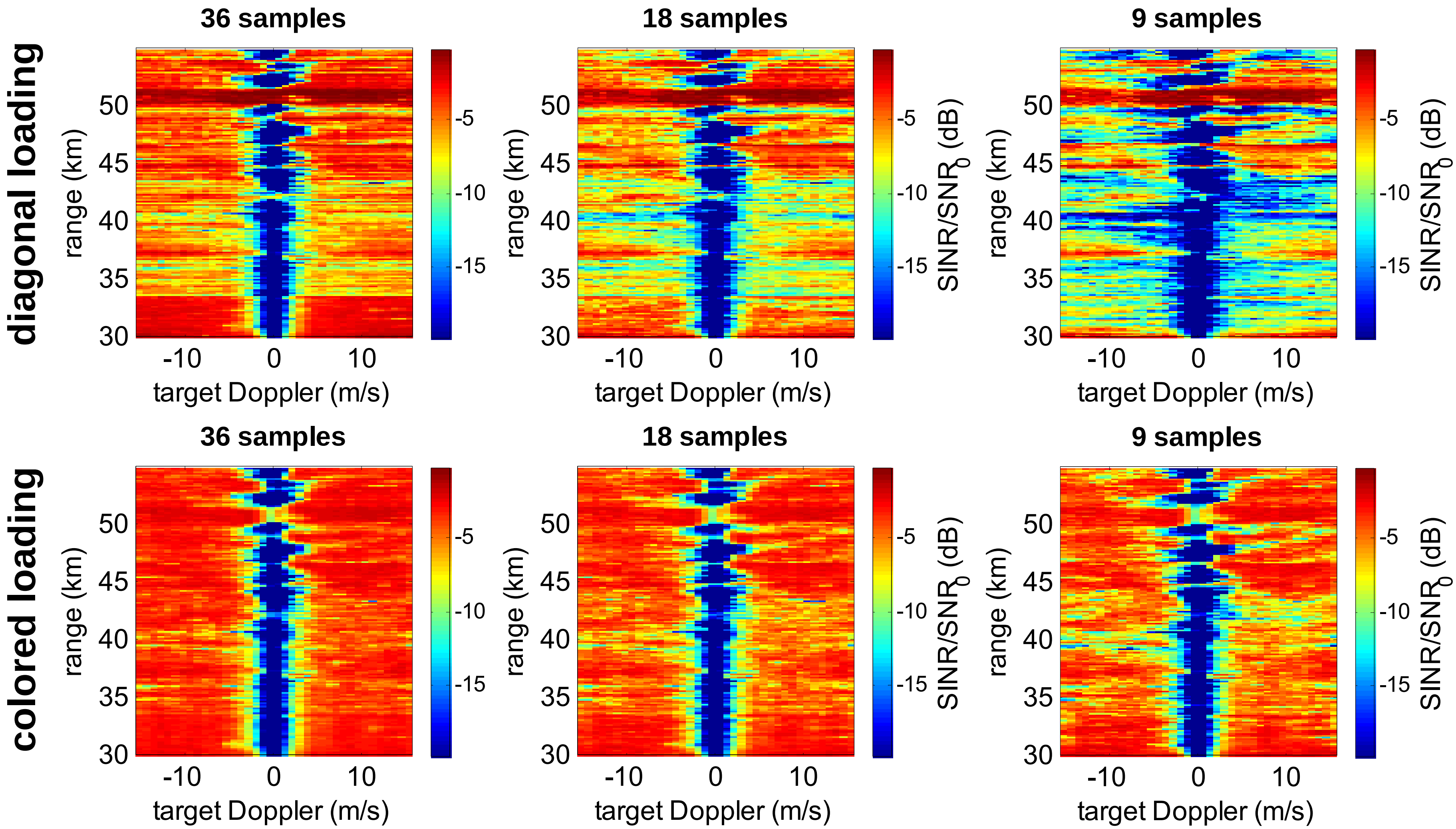
Traditional STAP





- **Comparison of ideal covariance SINR loss (left) and "traditional" STAP SINR loss (right)**
- **STAP results in wider clutter notch → poorer MDV if training window is wide compared to clutter "features"**

KASSPER Data Set 2



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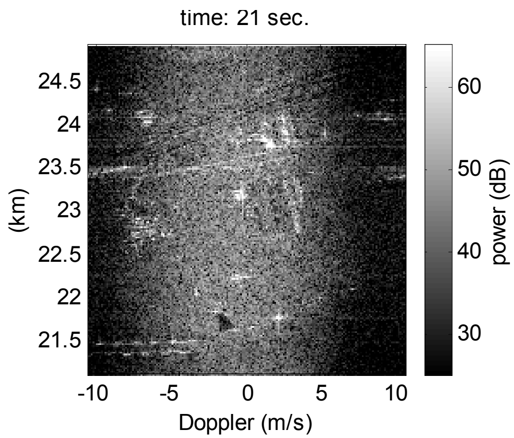
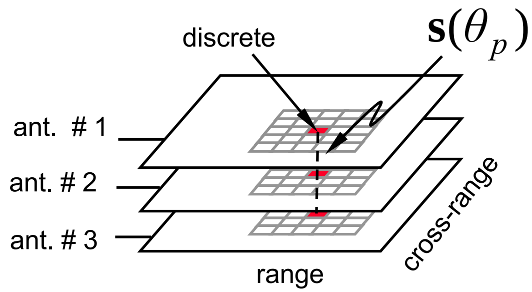
Discrete Clutter Colored Loading

STEP 1



threshold "low resolution" SAR map to detect discrete clutter

STEP 2



$$\mathbf{v}_m(\theta_p, f_p) = (\mathbf{H}_m^H \mathbf{t}(f_p)) \otimes \mathbf{s}(\theta_p)$$

$$\mathbf{t}_{[m]}(f_p) = \exp(j 2\pi m f_p T_{pri})$$

form space-time response for each discrete and transform to post-Doppler space (use observed spatial response)

STEP 3

$$\mathbf{Q}_{m,l} = \sum_{p=1}^{P_c} \mathbf{v}_{m,l}(\theta_p, f_p) \mathbf{v}_{m,l}^H(\theta_p, f_p)$$

$$\mathbf{R}_{m,l} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}_{m,k} \mathbf{x}_{m,k}^H + \beta \mathbf{Q}_{m,l} + \mathbf{I}$$

use responses to form a range-dependent "loading" matrix for each Doppler bin, add to sample cov. and run STAP processor

- **0.5 second CPI used to form "low resolution" SAR for discrete detector**
- **Final STAP processing w/ colored loading is performed on 100 ms CPI**
- **Discrete amplitudes not used in current implementation**
- **Colored loading level set to ~20 dB re. thermal noise**
- **Diagonal loading level set to ~0 dB re. thermal noise**

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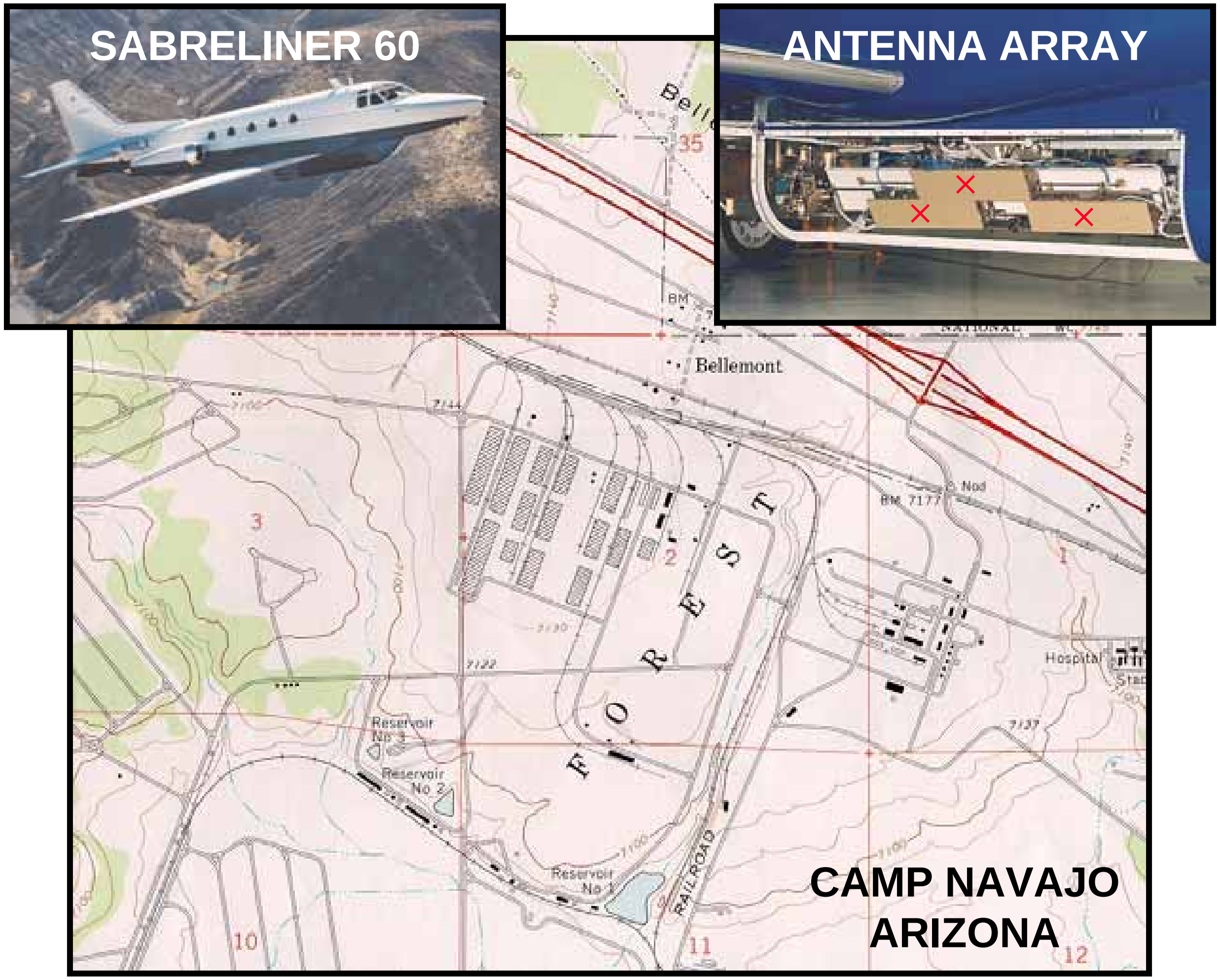
Tuxedo Data

Recorded Data



System Parameters for GMTI Mode

Center Freq.	9.6 GHz
Bandwidth	66 MHz
PRF	1,383 Hz
Tx Apertures	1
Rx Apertures	3
Az BW, Aperture	3.6 deg
El BW, Aperture	9.1 deg
Polarization	HH
A/C Heading	290 deg
Depr. Angle	15 deg
Recorded Time	30 sec

Post-Doppler, PRI-Staggered STAP:

Staggers	2
Stagger Offset	3 Pulses

CPI: 100 ms (128 pulses)

Limited targets in data (up to 5) and uniform terrain type (desert)

Baseline Processing

Conventional **STAP**

- **STAP**
  - 3 bin post-Doppler
  - element space
  - 200 training bins
  - 3 guard bins on each side
  - diagonal loading
- **Three truth targets provided with data**
  - circle: HMMWV
  - square: fuel truck
  - triangle: 5-ton truck
- **Additional targets observed**
- **Significant under-nulled clutter observed due to discretes**

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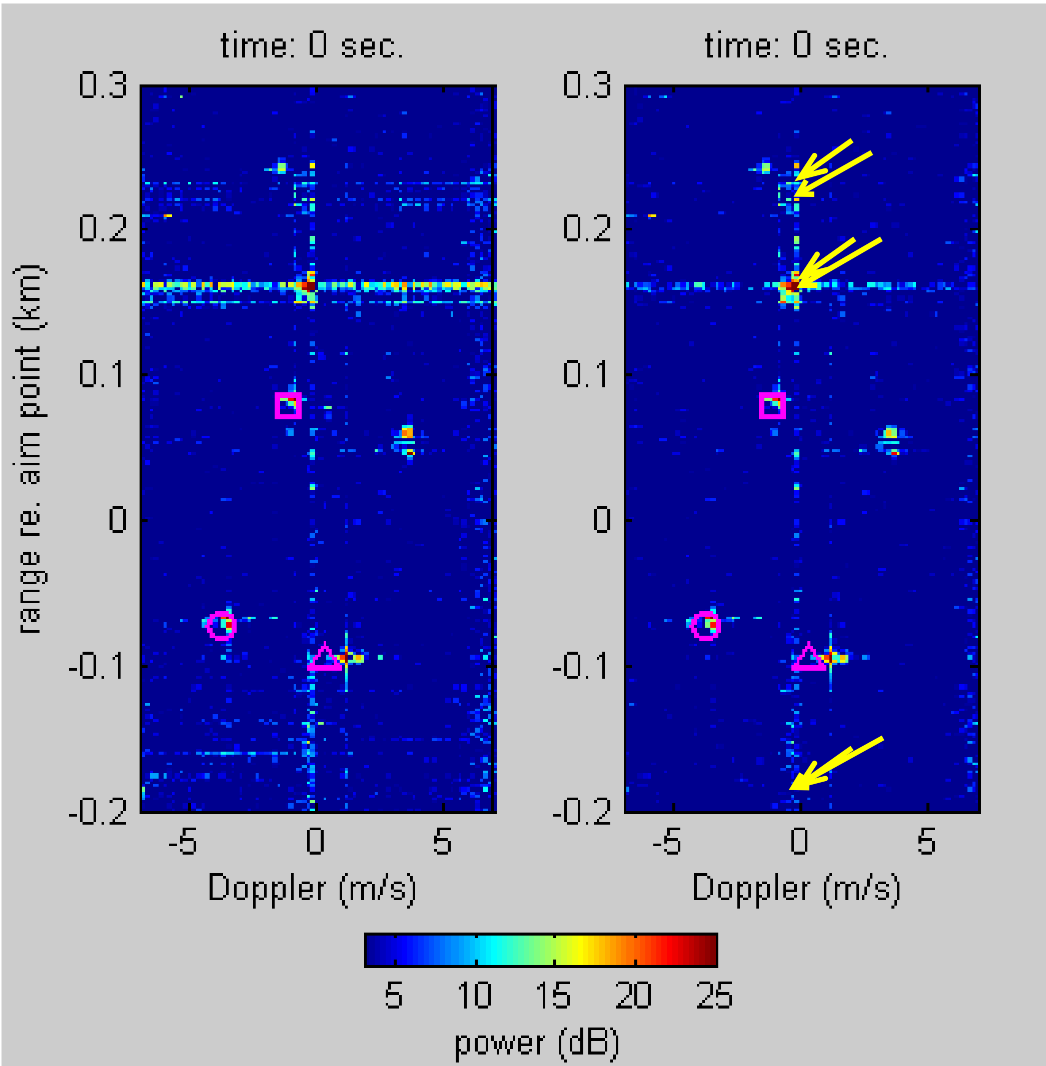
Click on Image to Play

KA-STAP Processing

STAP KA- STAP



Click on Image to Play

- STAP parameters same as previous slide
- KA-STAP also uses 200 training samples →
- Discrete threshold is ~30 dB re. thermal noise
- Discretes limited to region around zero Doppler (-0.5 m/s to +0.5 m/s)
- KA technique results in significant reduction of under-nulled clutter
- KA-STAP does not appear to impact moving target responses (i.e., pd is maintained)
- Yellow arrows mark locations of observed large clutter discretes

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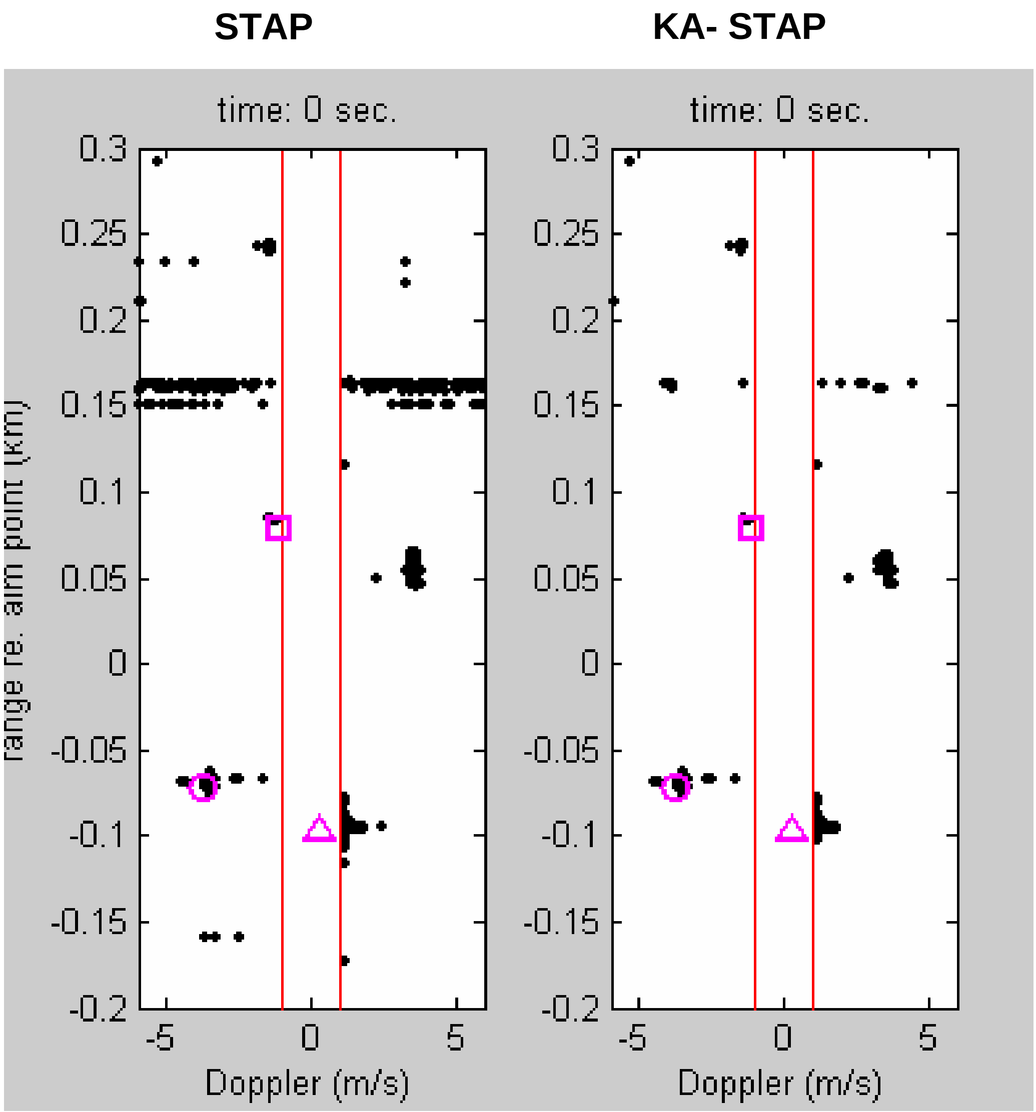
Raw Detections



Click on Image to Play

- **STAP weights normalized to give unit gain on thermal noise**
- **Constant threshold set at 10 re. thermal noise (no CACFAR)**
- **Detections for 3 CPIS shown in each movie frame**
- **No detections recorded between -1 m/s and 1 m/s**
- **KA technique results in fewer false alarms due to under-nulled clutter**
- **KA-STAP does not impact moving target responses (i.e., pd is maintained)**

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Composite Colored Loading

- **Colored loading can be readily applied to simultaneously address contributions due to distributed and discrete clutter → add two loading matrices**

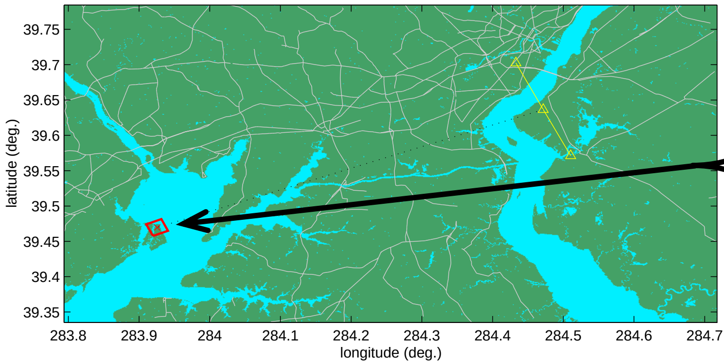
$$\mathbf{w}_m = \kappa\left(\mathbf{R}_{s,m} + \beta_{L,m}\mathbf{I} + \beta_{d_1,m}\mathbf{R}^1_{c_1,m} + \beta_{d_2,m}\mathbf{R}_{c_2,m}\right)^{-1}\mathbf{v}_m$$

distributed clutter

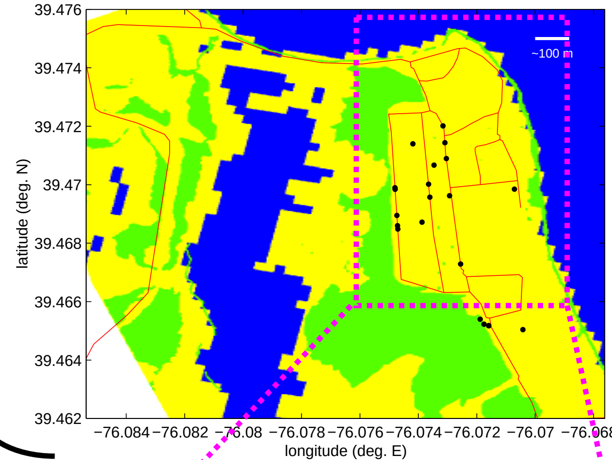
discrete clutter

- **Discrete clutter component is generally range dependent → will require updating the weights for each discrete to be nulled**

KASSPER Data Set 3 Overview

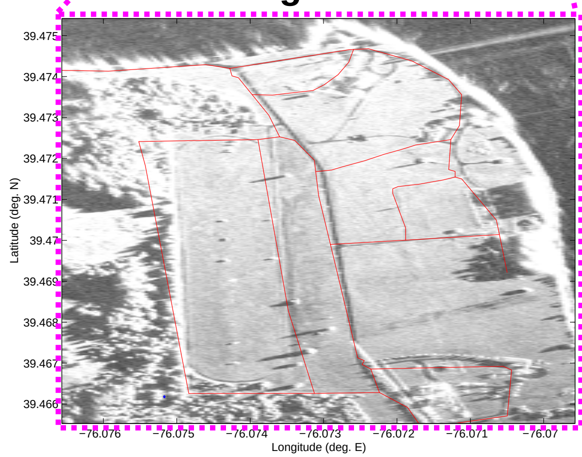


land cover, roads, targets



- **30 CPIs spanning 2.5 minutes (5 sec. revisit)**
- **Ground clutter characterized by geo-coded SAR images provided by ALPHATECH**
- **USGS land cover used for ICM**
- **22 slow-moving ground targets traveling on and off roads**

SAR image w/ roads



Freq.	X-band
BW	20 MHz
PRF	1 kHz
Pulses	64
Ant.	3.5 m x 0.3 m
ICM	Billingsley (15 m.p.h.)

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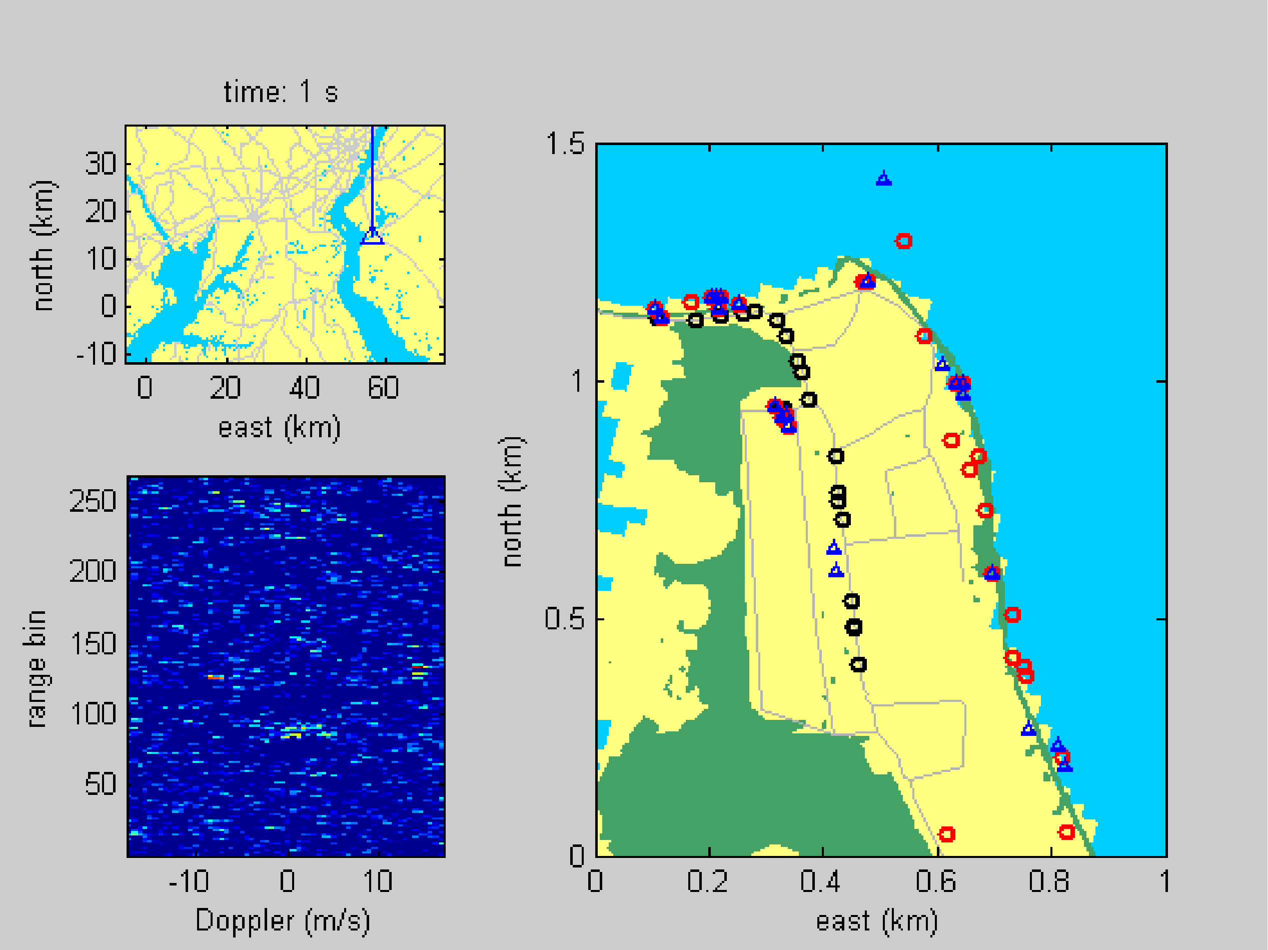
KASSPER Data Set 3

Click on Image to Play



- **Upper left: wide-area view with platform and scanning angle**
- **Lower left: range-Doppler beamformer output after CFAR normalization**
- **Right: plan view with land cover, truth and detections**
  - Land cover
    - » Blue: water
    - » Green: trees
    - » Yellow: grass
    - » Gray lines: roads
  - black: truth
  - blue: STAP w/ CL
  - red: STAP

NOTES:

- 3 bin post-Doppler element space STAP
- 36 training samples – excluding bin under test
- Clutter-only data used for training (i.e., no target contamination)
- Full-DoF colored loading level set to 20 dB re. noise
- Diagonal loading set to 3 dB re. noise
- 15 dB threshold re. noise

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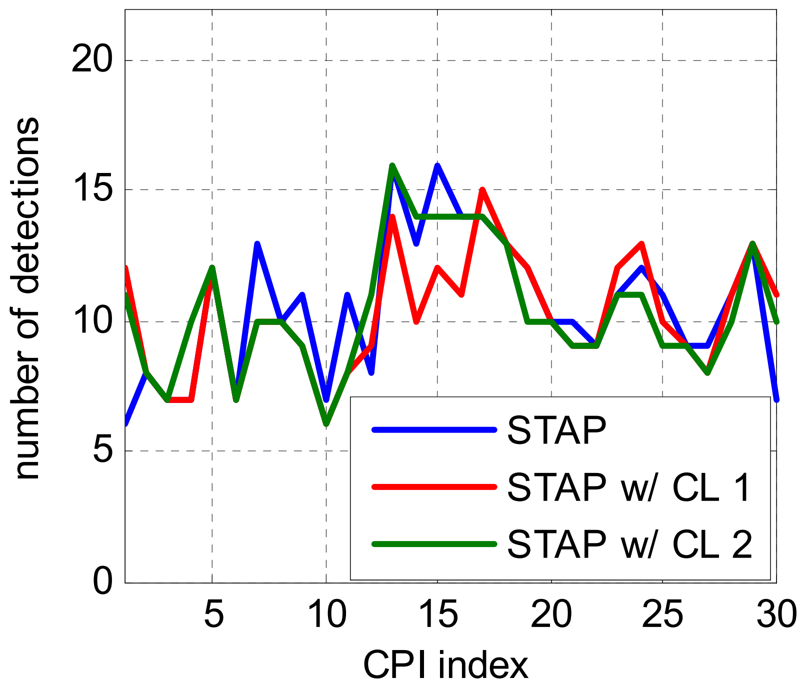
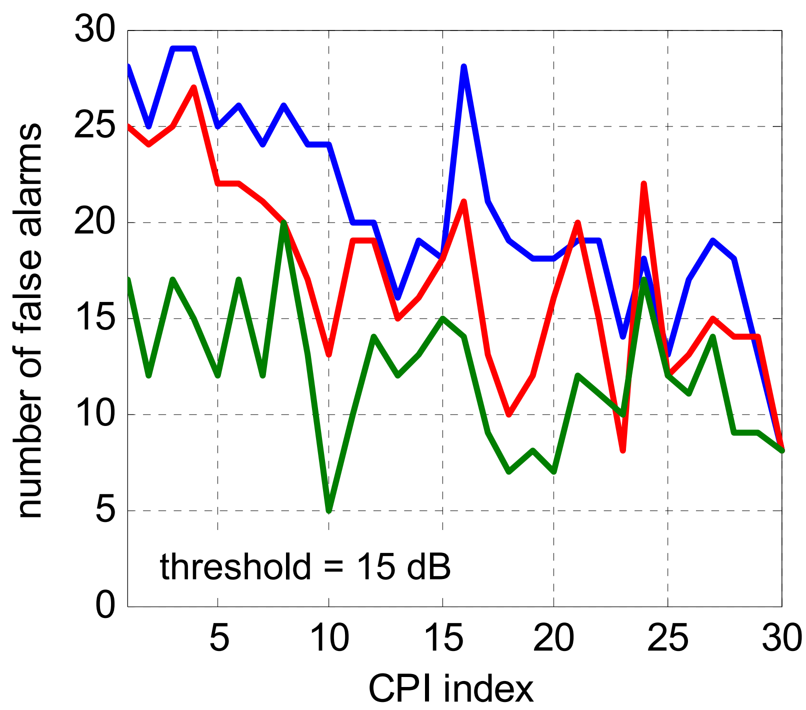
Detection Summary





- **Truth target declared detected when there is at least one detection that associates (22 total truth targets)**
- **Detections that do not associate with a truth target are considered false alarms**
- **Results shown for distributed clutter colored loading ('STAP w/ CL 1') and composite colored loading ('STAP w/ CL 2)**
- **The 10 largest discretes in the conventional beamformer range-Doppler clutter map are used in the composite CL model**

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Data Domain STAP

- **Problem is to solve linear system of equations to get the weight vector:**

$$\mathbf{w} = \mathbf{R}_s^{-1}\mathbf{s} \leftrightarrow \mathbf{R}_s\mathbf{w} = \mathbf{s}$$

- **Since the sample covariance is typically Hermitian and positive definite one approach is to find its Cholesky decomposition**

$$\mathbf{C}^H\mathbf{C}\mathbf{w} = \mathbf{s}$$

- **The matrix C is upper triangular so the solution can be readily found using a forward and backward substitution**

$$\mathbf{C}^H\mathbf{y} = \mathbf{s} \rightarrow \mathbf{C}\mathbf{w} = \mathbf{y}$$

Data Domain STAP (cont.)

- It turns out that C can be found without forming the sample covariance by performing a QR decomposition (Q unitary, R upper triangular) on the data matrix directly

$$\mathbf{X}^H = \mathbf{QR} \Rightarrow \mathbf{R}_s = \mathbf{XX}^H = (\mathbf{QR})^H(\mathbf{QR}) = \mathbf{R}^H\mathbf{R}$$

- In this case R is upper triangular and equivalent to the Cholesky decomposition of R_s so it can readily be used to solve for the weight vector (see previous slide)
- It is possible to compute R without explicitly finding Q which saves significant computations
- Diagonal loading can be included by simply augmenting the data matrix

$$\mathbf{X}_a = \begin{bmatrix} \mathbf{X} & \sqrt{\beta}\mathbf{I} \end{bmatrix} \Rightarrow \mathbf{R}_{s,L} = \mathbf{X}_a\mathbf{X}_a^H = \mathbf{R}_s + \beta\mathbf{I}$$

- In general any solution that adds a positive semi-definite matrix to the covariance can be implemented by an appropriate augmentation of the data matrix in the data domain implementation

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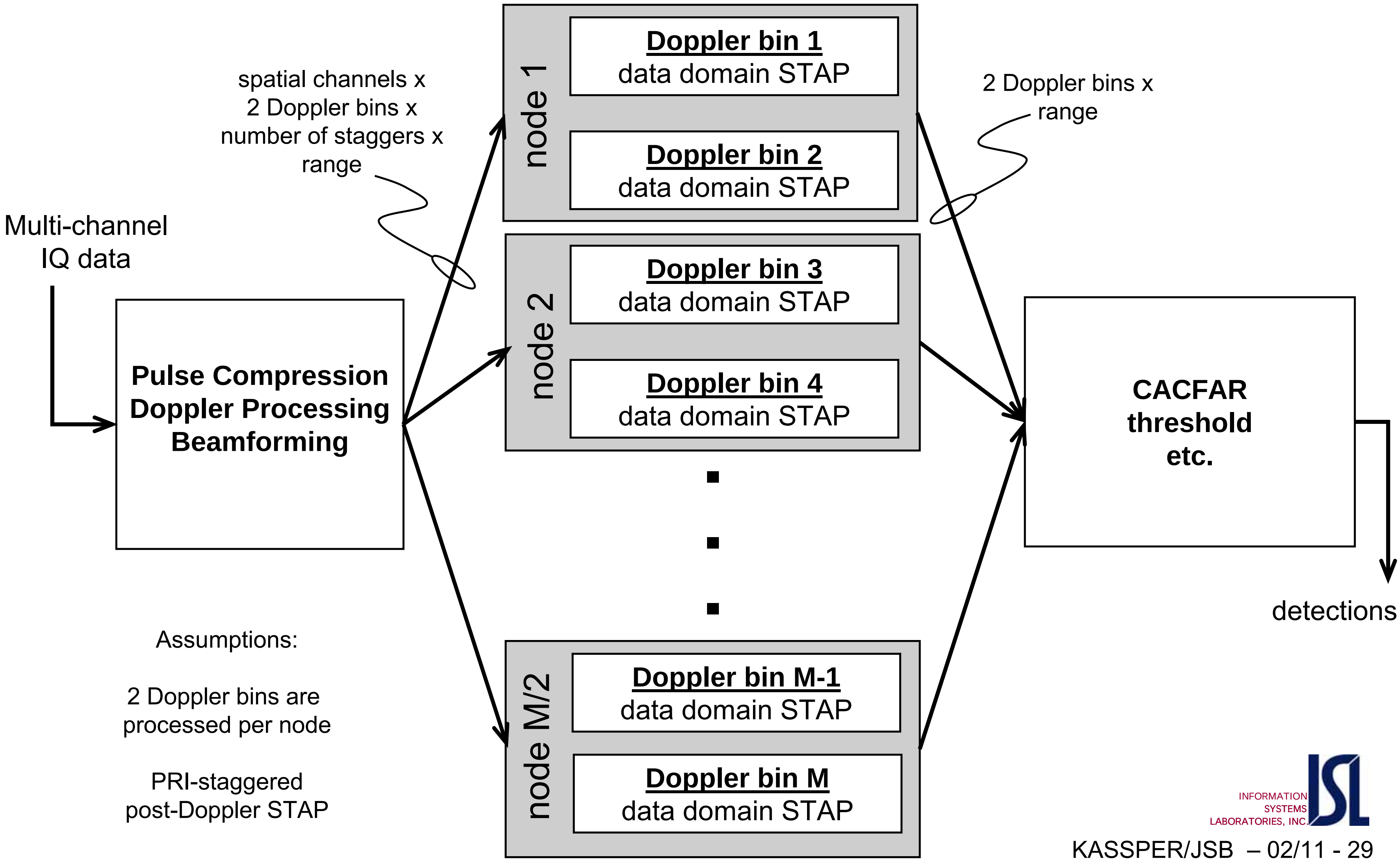
STAP Algorithm Partitioning Example



Assumptions:

2 Doppler bins are processed per node

PRI-staggered post-Doppler STAP

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Data Domain Colored Loading

- **Typically, the loading matrix Q_m is Hermitian and positive-definite so that its Cholesky decomposition exists**

$$\mathbf{Q}_m = \beta_\mathrm{d}\mathbf{R}_{c,m} + \beta_\mathrm{L}\mathbf{I} = \mathbf{Q}_m^{1/2}\mathbf{Q}_m^{1/2} = \mathbf{C}_m^H\mathbf{C}_m$$

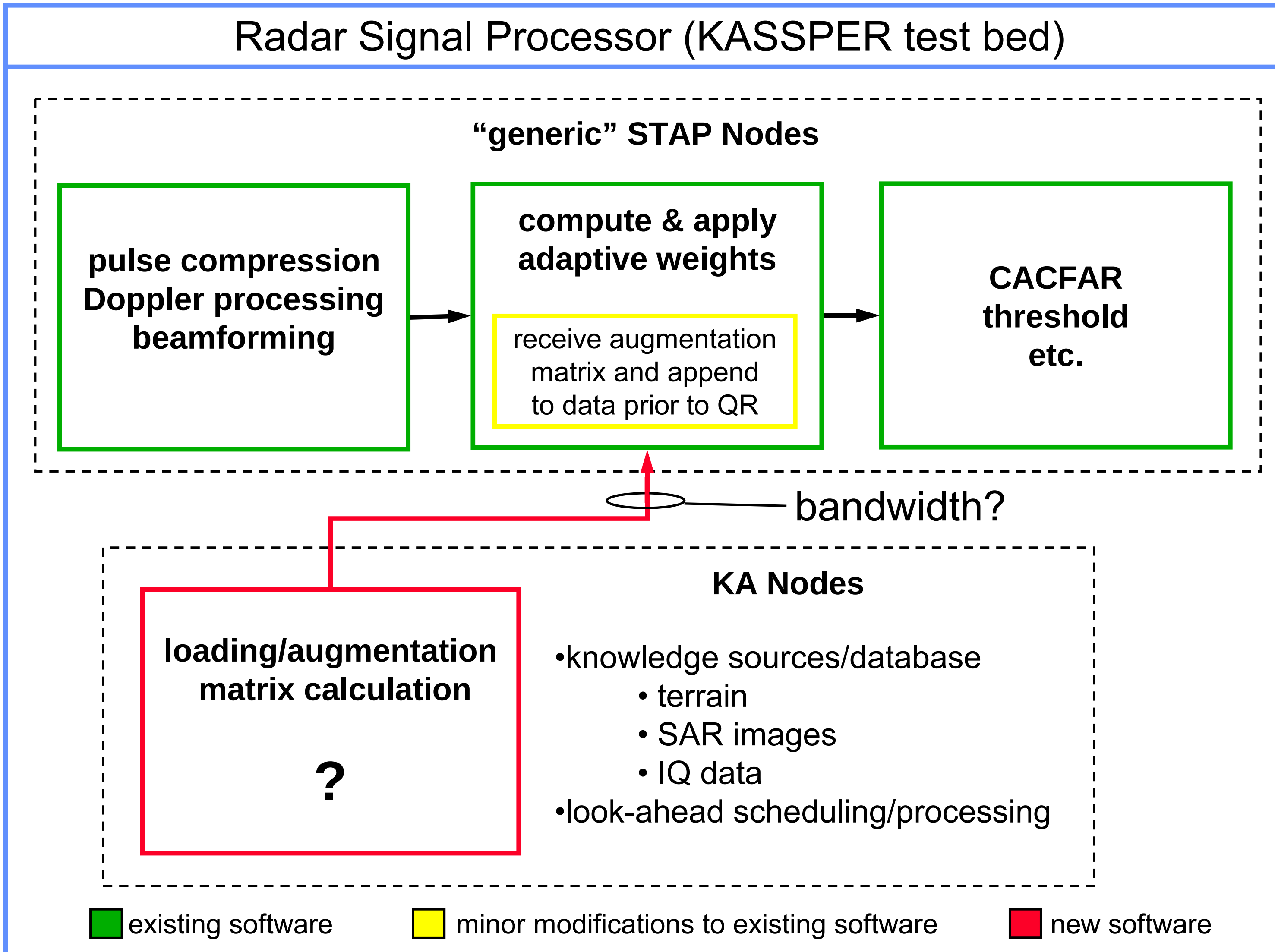
$\mathbf{C}_m$ is upper triangular

- **Can be efficiently implemented in the data domain by augmenting data matrix and using QR decomposition**

$$\mathbf{X}_a = [\mathbf{X}_m \quad \sqrt{K}\mathbf{C}^H] \rightarrow \frac{1}{K}\mathbf{X}_a\mathbf{X}_a^H = \mathbf{R}_s + \mathbf{Q}_m$$

- **Thus, pre-filter approach implemented as colored loading actually "fits" existing STAP computing architectures (if they already employ data domain processing with diagonal loading)**

Real-Time Implementation Approach



Bandwidth Requirements

- **Assumptions:**
  - One complex *D x D* loading/augmentation matrix is required per Doppler bin per CPI
    - » In covariance domain matrix is Hermitian symmetric
    - » In data domain matrix is all zeros below main diagonal
    - » Only need to send upper (or lower) triangular part of the matrix
  - 128 Doppler bins
  - 100 ms CPI length
  - 2 staggers, 3 beams (D = 6)
  - Double precision real and imaginary data
- **Bandwidth required to communicate augmentation matrices**

$$BW = \frac{(128\text{bits})(128)(D(D+1)/2)}{100\text{ms}} = 3.4\text{Mbps}$$

- **May want to only apply this technique to a small number of Doppler bins**

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Real-Time Development Tasks

- Set up PVL development environment on ISL Linux cluster
- **Obtain existing KASSPER real-time testbed STAP source code from MIT Lincoln Laboratory → load, build, and execute using simulated data inputs (ISL received source code 01/05)**
- **Integrate ISL colored loading technique**
  - Choose a colored loading matrix and test with PRI staggered post-Doppler STAP (MATLAB)
  - Code the colored loading matrix calculation in PVL/VSIPL
  - Integrate with STAP code (test and compare with MATLAB version)
- **Demonstrate parallel algorithm on ISL Linux cluster**
  - Define mapping across nodes
  - Execute with simulated and experimental data
  - Test and compare with MATLAB implementation
- **Port to MIT Lincoln Laboratory cluster**
- **Port to KASSPER Mercury test best system**
- **Documentation**

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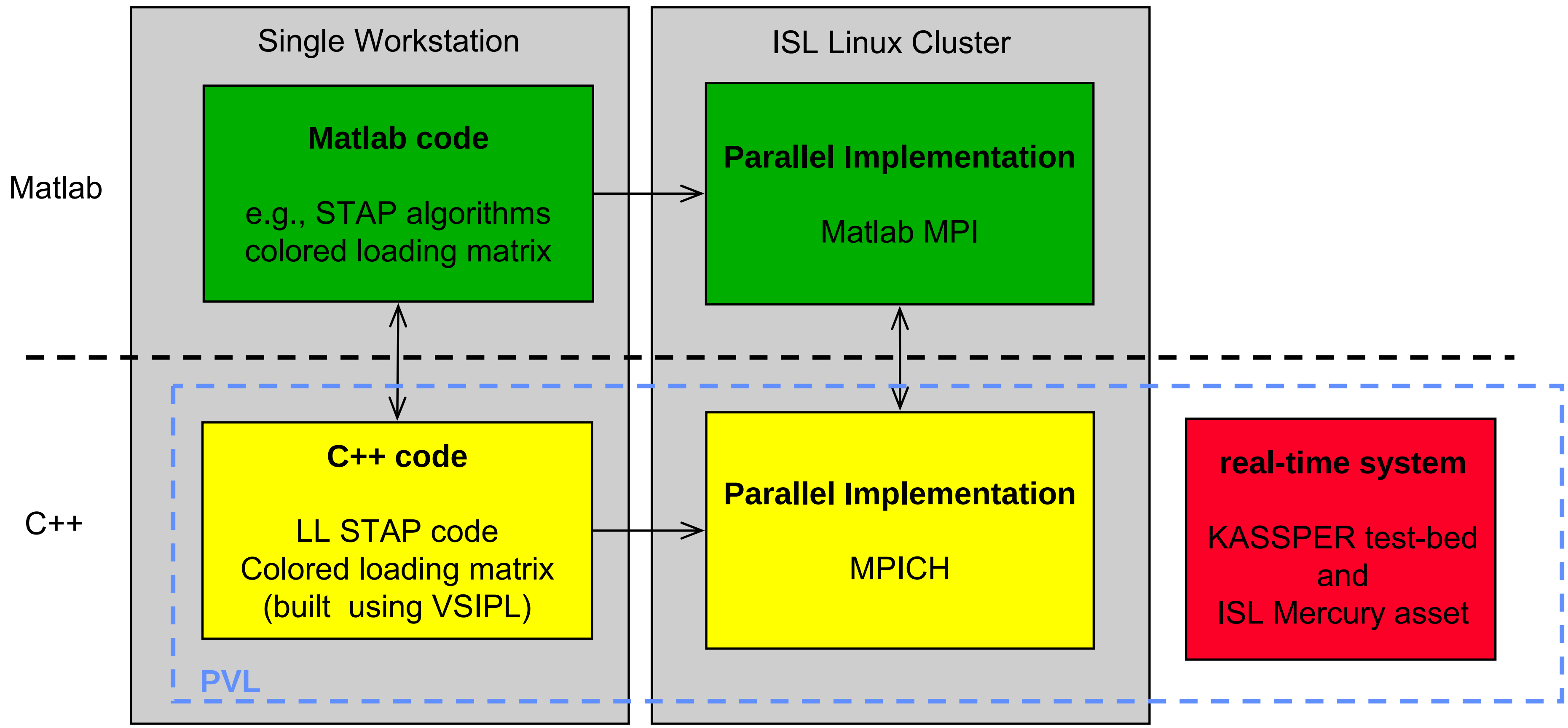
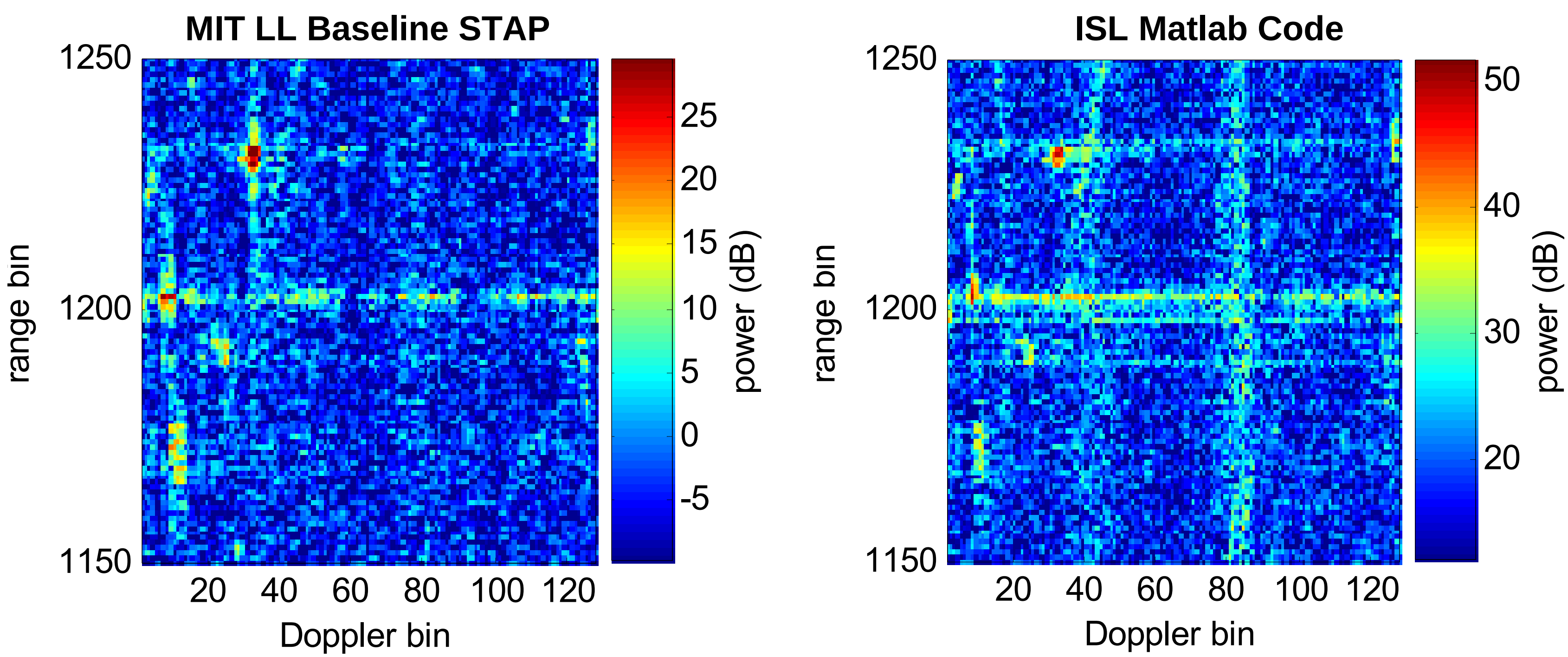
Software Development Environment

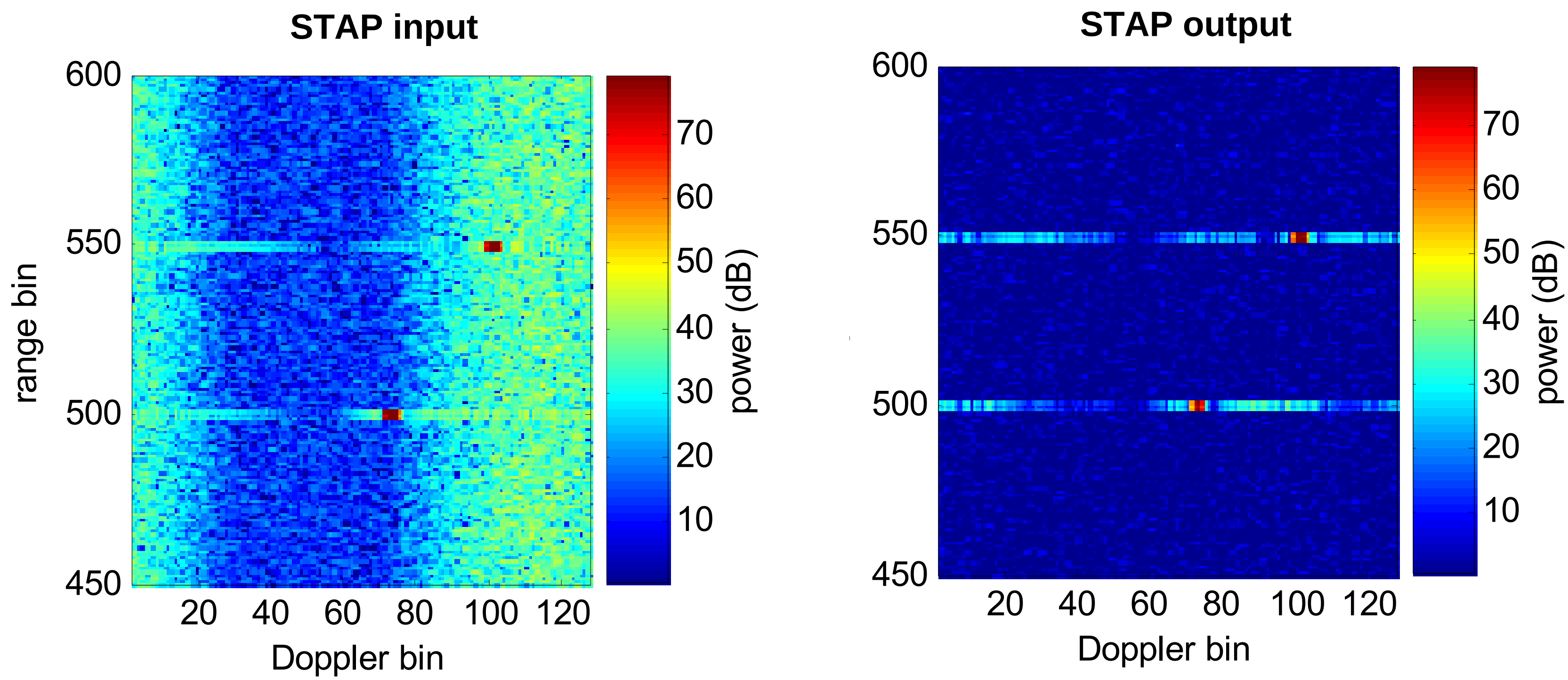


Tuxedo Processing Example



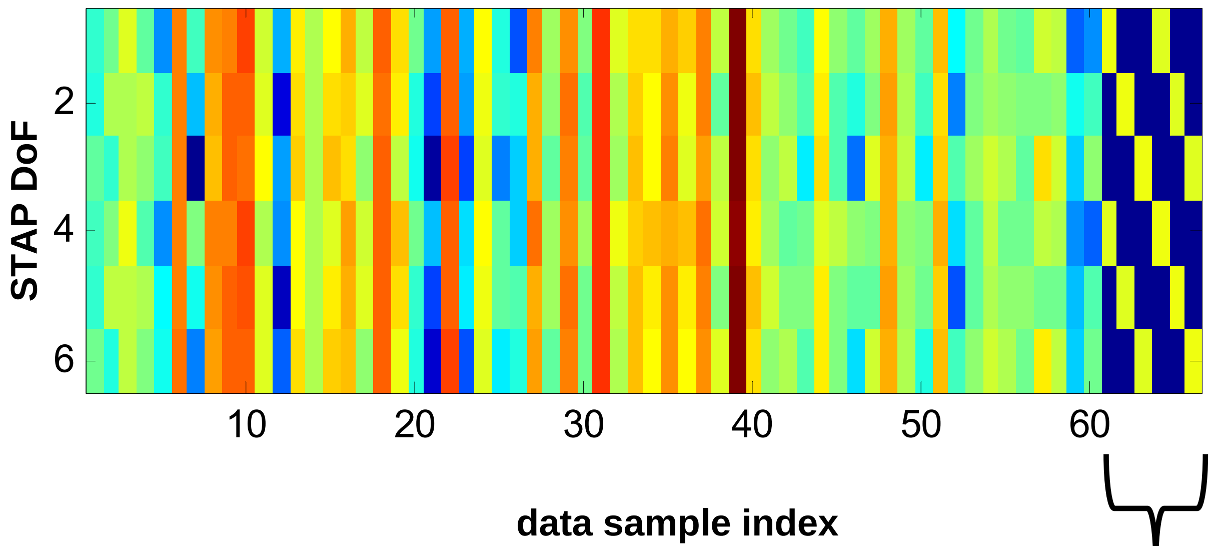
- **Comparison of Tuxedo data processed using KASSPER baseline STAP code (C++) and ISL Matlab STAP algorithm**
  - KASSPER code is PRI-stagger post-Doppler, ISL code is multi-bin post Doppler
  - KASSPER code is AMF normalized, ISL code is normalized to unit gain on white noise
  - Good match between major beamformer residue features
- **Demonstrates we can build and run the code and understand and manipulate the output**

Simulated Data Processing Example



- **A simulated datacube containing clutter and two test targets with same parameters as the Tuxedo data was generated and used as input to the KASSPER code**
- **Demonstrates ability to manipulate the code inputs**
- **Next step is to input KASSPER simulated data sets (e.g., KASSPER data sets 2 and 3)**

Range Independent Loading



- **Matrix augmented to account for diagonal loading**
- **Not an identity matrix due to correlation between adaptive channels in PRI-stagger algorithm**
- **Baseline code actually implements "colored" loading → one matrix only, not updated on each CPI**

- **Example data matrix amplitude output from the KASSPER code for a single Doppler bin shown**
- **6 DoFs**
  - 3 spatial
  - 2 temporal
- **60 training samples**
- **Augmented matrix to account for "diagonal" loading visible**

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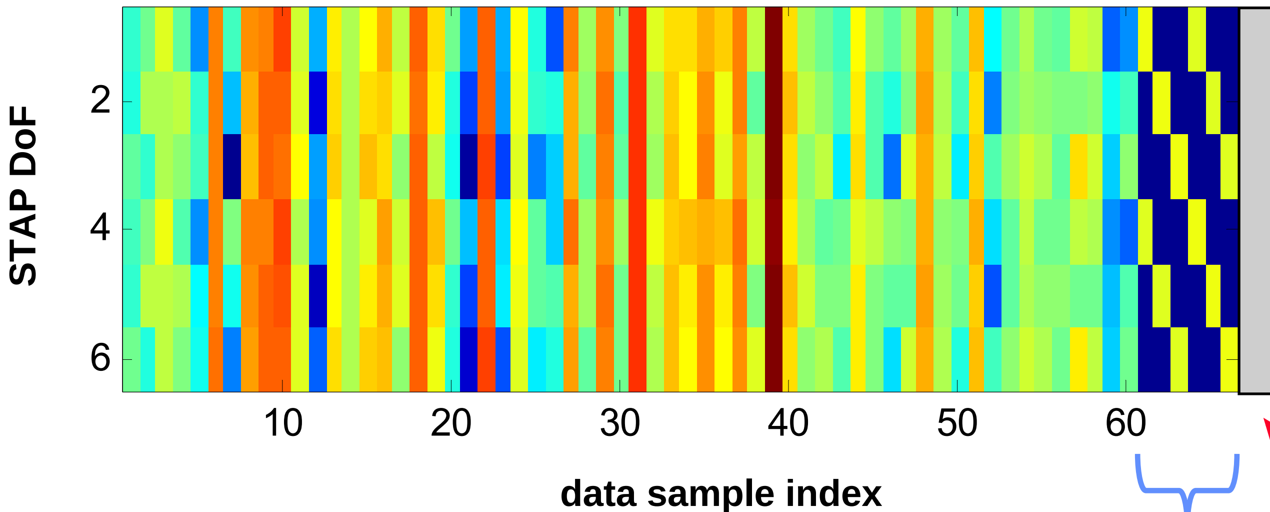
Range Dependent Loading



insert distributed clutter loading matrix here

insert small number of vectors to account for each discrete clutter patch → may require QR updating to be efficient

Outline

- **Background**
- **Knowledge-Aided Data Pre-Whitening**
- **Example Applications**
  - Distributed clutter
  - Discrete clutter
  - Combined discrete and distributed clutter
- **Real-Time Implementation Analysis**
  - Implementation approach
  - Real-time demonstration status
- **Future Plans and Summary**

Future Plans and Summary

- Colored loading technique has been tested on several simulated and one experimental data set (Tuxedo)
- The technique typically leads to fewer false alarms for cases when low sample support is required
- Combining colored loading with information derived from conventional beamformer output (or low resolution SAR) shown to help mitigate under-nulling of strong discrete clutter
- Future work will focus on detailed analysis/simulation of loading levels→ develop rules for setting colored loading levels
- Demonstration of colored loading performance will continue using additional simulated data sets and experimental data → movies showing performance over time
- Task to implement colored loading on the KASSPER test-bed is underway, including initial evaluation of the existing C++ KASSPER baseline STAP algorithm written using PVL

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Backups

Linear Constraints

- **Re-write quadratic constraint using the eigen-decomposition of the *a priori* clutter model, $\mathbf{R}_c = \mathbf{U}^H\mathbf{D}\mathbf{U}$ (dominant subspace)**

$$\mathbf{w}^H\mathbf{R}_c\mathbf{w} = 0 \Rightarrow \mathbf{w}^H(\mathbf{U}\mathbf{D}\mathbf{U}^H)\mathbf{w} = 0 \Rightarrow (\mathbf{w}^H\mathbf{U})\mathbf{D}(\mathbf{U}^H\mathbf{w}) = 0$$

$$\Rightarrow \mathbf{w}^H\mathbf{U} = \mathbf{0} \quad \because \mathbf{D}$$

has strictly positive diagonal elements (also has dimensions << $\mathbf{R_c}$ e.g., Brennan's Rule)

- **A set of linear constraints**

$$\min_{\mathbf{w}} \quad E\{|\mathbf{w}^H\mathbf{x}|^2\} \quad \text{s.t.} \quad \mathbf{w}^H\mathbf{v} = 1$$

$$\mathbf{w}^H\mathbf{U} = \mathbf{0}$$

$$\mathbf{w}^H\mathbf{w} = \delta$$

desire weights to be orthogonal to *a priori* clutter model

this is the KA part

- **Gives:**

$$\mathbf{w} = \frac{\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{U})^{-1}\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1})\mathbf{v}}{\mathbf{v}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{U})^{-1}\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1})\mathbf{v}} = \frac{\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{P}\mathbf{v}}{\mathbf{v}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{P}\mathbf{v}}$$

$$\overline{\mathbf{R}}_{\mathbf{xx}} = \mathbf{R}_{\mathbf{xx}} + \beta_L\mathbf{I}$$

Quadratic vs. Linear Constraints

- **The two solutions:**

$$\mathbf{w} = \frac{(\overline{\mathbf{R}}_{\mathbf{xx}} + \beta_d \mathbf{R}_c)^{-1}\mathbf{v}}{\mathbf{v}^H(\overline{\mathbf{R}}_{\mathbf{xx}} + \beta_d \mathbf{R}_c)^{-1}\mathbf{v}} \qquad \mathbf{w} = \frac{\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{P}\mathbf{v}}{\mathbf{v}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{P}\mathbf{v}}$$

- **Manipulation of the quadratic constraint solution using the matrix inversion lemma and eigen-decomposition highlights the difference between the two solutions**

$$\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{P} = \overline{\mathbf{R}}_{\mathbf{xx}}^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{U})^{-1}\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1})$$

$$(\overline{\mathbf{R}}_{\mathbf{xx}} + \beta_d \mathbf{R}_c)^{-1} = \overline{\mathbf{R}}_{\mathbf{xx}}^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{U} + \boxed{\frac{1}{\beta_d}\mathbf{D}^{-1}})^{-1}\mathbf{U}^H\overline{\mathbf{R}}_{\mathbf{xx}}^{-1})$$

- **The two solutions are equivalent in the limit of infinite loading and/or large clutter model eigenvalues**
- **Since the linear constraints cause the weights to be precisely orthogonal to the known covariance the quadratic constraint achieves this constraint only approximately**