Reduced Degree-of-Freedom STAP with Knowledge-Aided Data Pre-Whitening

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INFORMATION
SYSTEMS
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Outline

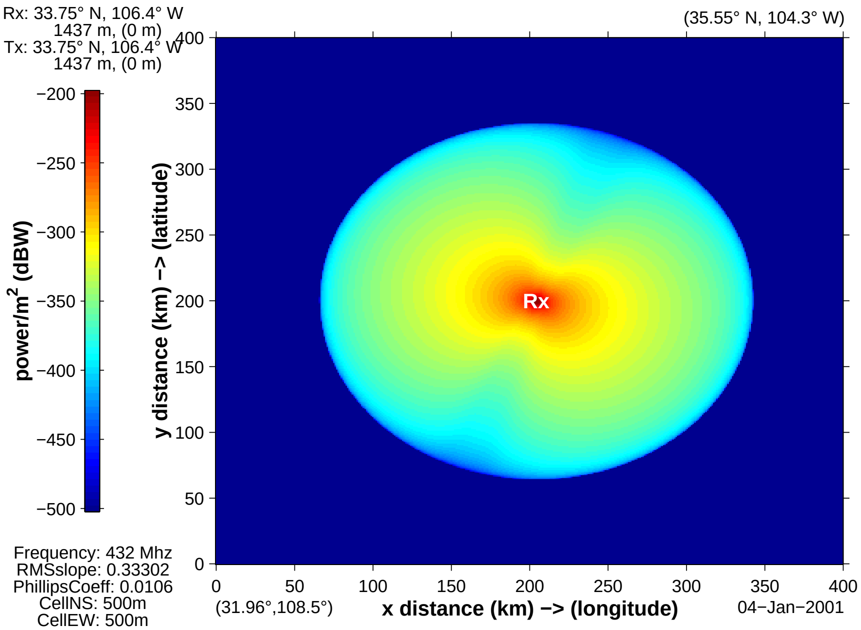
- **Background**
- **Algorithm Development**
- **Performance Results**
- **Summary**

Background

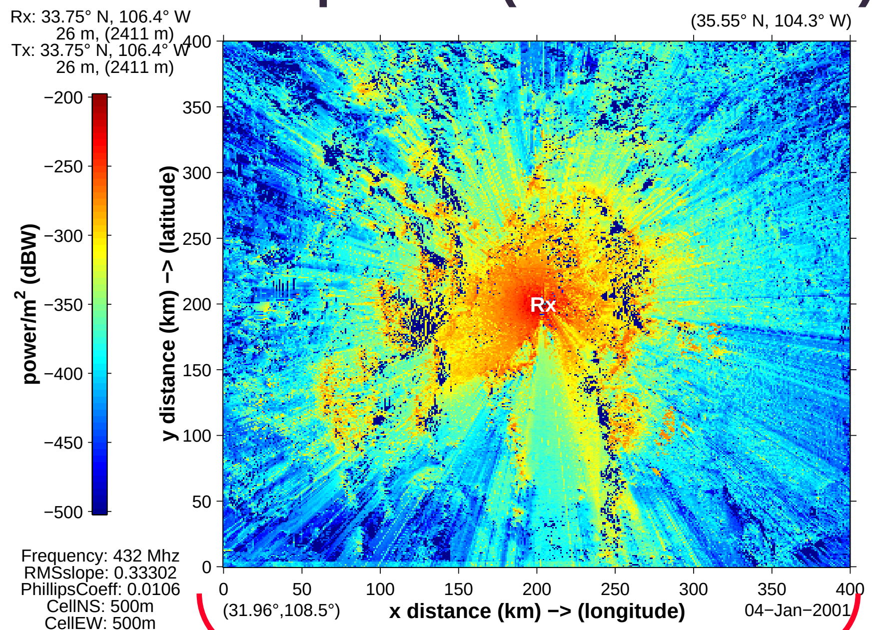
- **Real-world radar clutter environments depend on site-specific factors including:**
  - Terrain
  - Ground cover type
- **Site-specific clutter modeling is fundamental to understanding STAP performance in real-world settings**
- **This has led to the development of site-specific performance bound techniques**
  - Thermal noise limited performance is optimistic for systems operating in real-world environments
  - Theory is based on ideal site-specific clutter covariance modeling
- **It is logical that the models used in site-specific performance analyses could also be used when processing the radar data to potentially improve radar performance**
- **For many systems it is not possible to use full-DoF STAP due to limited computational resources and limited sample support**

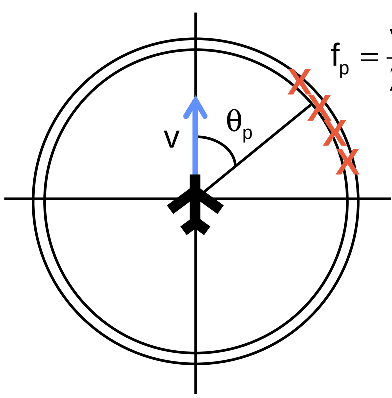
Site-specific Clutter Modeling

bald earth



site-specific (SCATS/DTED)





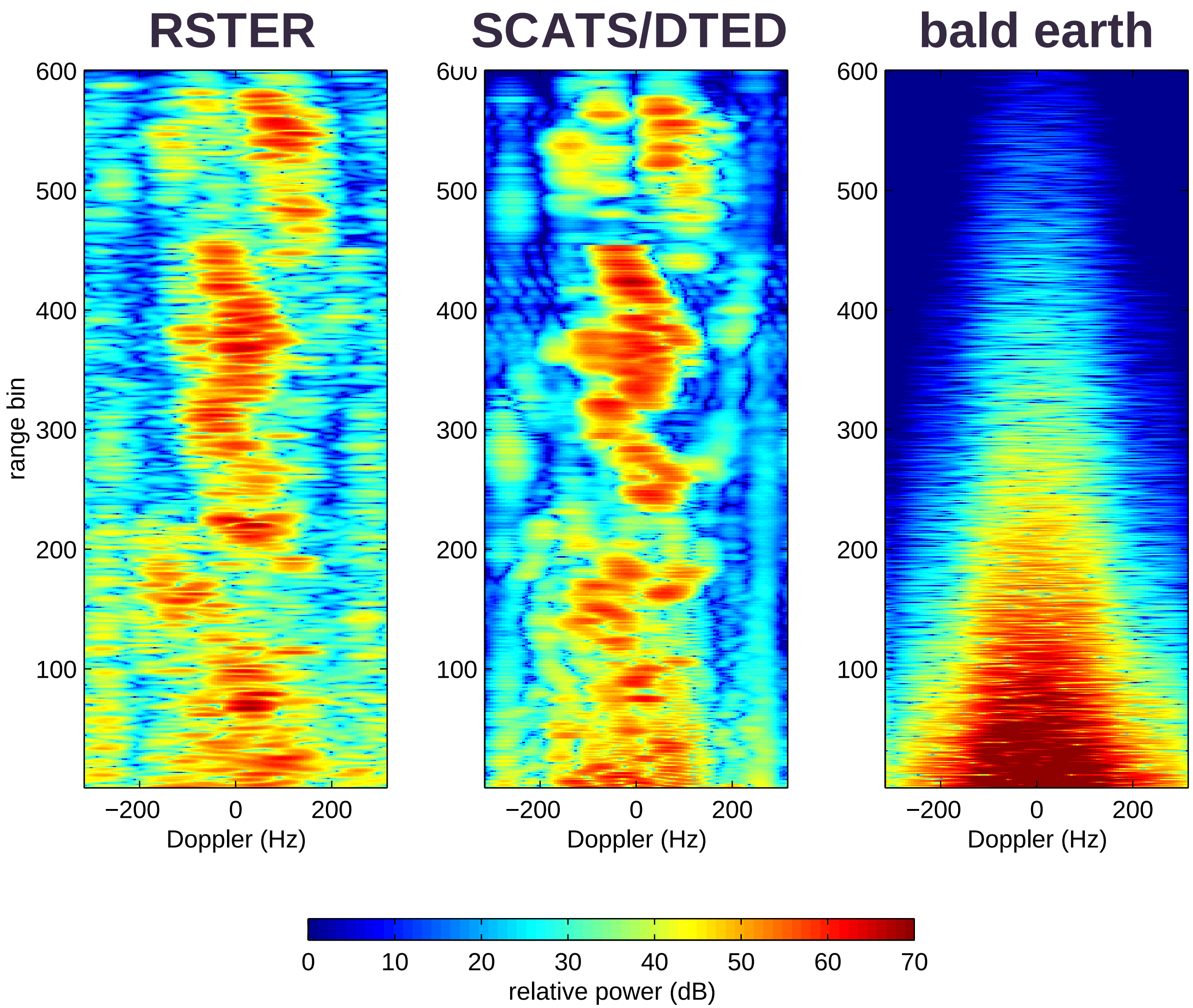
$$f_p = \frac{v}{\lambda}\cos(\theta_p)$$

ICM
calibration errors
channel mismatch

$$\mathbf{R}_c = \sum_{p=1}^{P_c} |\alpha_p|^2 \mathbf{v}(\theta_p, f_p)\mathbf{v}^H(\theta_p, f_p) \circ \mathbf{T}_p$$

sum over all scattering patches in a given range bin

Mountain Top Monostatic Clutter



- Range-Doppler clutter maps shown for RSTER and simulations
- Simulation results shown both with and without DTED
- Simulation w/ DTED results in a significantly better characterization of the experimental data
- Site-specific models capture a majority of the clutter features

Knowledge-Aided Signal Processing

- **The *a priori* knowledge will typically be used in two ways**
  - Indirect: exploit knowledge sources to segment training data, etc.
  - Direct: exploit knowledge sources to place nulls in the beamformer pattern
- **This presentation develops a methodology for using *a priori* knowledge *directly* in the reduced-DoF space-time beamforming solution**
- **Clutter cancellation based on *a priori* knowledge alone will typically not result in adequate performance**
- **Focus will be on techniques that combine or “blend” adaptive and deterministic filtering**
- **The performance of these filtering techniques will be a function of how well the system is calibrated**

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Interference Modeling

- Assume the clutter signal plus thermal noise model

$$\mathbf{x} = \mathbf{x}_c \circ \mathbf{t} + \mathbf{n}$$

$\circ$ - Hadamard product (element-wise product)

- The modulation will typically be small

$$\mathbf{t} = \mathbf{1} + \mathbf{d}$$

$\mathbf{d}$ is zero-mean, variance << 1

- Clutter signal with small modulation

$$\mathbf{x} = \mathbf{x}_c + \mathbf{x}_c \circ \mathbf{d} + \mathbf{n}$$

- Clutter correlation matrix

$$E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{R}_{\mathbf{xx}} = E\{\mathbf{x}_c\mathbf{x}_c^H\} + E\{\mathbf{x}_c\mathbf{x}_c^H\} \circ E\{\mathbf{d}\mathbf{d}^H\} + \sigma^2\mathbf{I}$$

$$= \underbrace{\mathbf{R}_c}_{\text{"known" component}} + \underbrace{\mathbf{R}_c \circ \mathbf{T} + \sigma^2\mathbf{I}}_{\text{unknown component}}$$

Knowledge-Aided Quadratic Constraints
Full-DoF STAP

- **The usual optimization problem:**

$$\min_{\mathbf{w}} \ E\{|\mathbf{w}^H\mathbf{x}|^2\} \quad \text{s.t.} \quad \mathbf{w}^H\mathbf{v}=1 \quad \rightarrow \quad \mathbf{w}=\frac{\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{v}}{\mathbf{v}^H\mathbf{R}_{\mathbf{xx}}^{-1}\mathbf{v}}$$

- **Incorporate covariance model as a quadratic constraint**

$$\min_{\mathbf{w}} \ E\{|\mathbf{w}^H\mathbf{x}|^2\} \quad \text{s.t.} \quad \begin{cases} \mathbf{w}^H\mathbf{v}=1 \\ \mathbf{w}^H\mathbf{R}_c\mathbf{w} \le \delta_d \\ \mathbf{w}^H\mathbf{w} \le \delta_L \end{cases}$$

want weights to be "orthogonal" to *a priori* clutter model

this is the KA part

- **Gives:**

$$\mathbf{w}=\frac{(\mathbf{R}_{\mathbf{xx}}+\beta_d\mathbf{R}_c+\beta_L\mathbf{I})^{-1}\mathbf{v}}{\mathbf{v}^H(\mathbf{R}_{\mathbf{xx}}+\beta_d\mathbf{R}_c+\beta_L\mathbf{I})^{-1}\mathbf{v}}=\frac{(\mathbf{R}_{\mathbf{xx}}+\mathbf{Q})^{-1}\mathbf{v}}{\mathbf{v}^H(\mathbf{R}_{\mathbf{xx}}+\mathbf{Q})^{-1}\mathbf{v}}$$

"colored loading"

Constraint Satisfaction

- **The two loading levels are determined by assuring satisfaction of the two soft constraints**
- **Leads to two coupled non-linear inequality relations for the two real scalar loading levels embedded in Q**

$$\mathbf{v}^H(\mathbf{R_{xx}}+\mathbf{Q})^{-1}\mathbf{R}_c(\mathbf{R_{xx}}+\mathbf{Q})^{-1}\mathbf{v}\le\delta_d\left(\mathbf{v}^H(\mathbf{R_{xx}}+\mathbf{Q})^{-1}\mathbf{v}\right)^2 \qquad (1)$$

$$\mathbf{v}^H(\mathbf{R_{xx}}+\mathbf{Q})^{-2}\mathbf{v}\le\delta_L\left(\mathbf{v}^H(\mathbf{R_{xx}}+\mathbf{Q})^{-1}\mathbf{v}\right)^2 \qquad (2)$$

- **No closed form solution, must be solved iteratively**
- **In the white noise gain relation, $\delta_L > 0$ to obtain solution**
- **In the clutter orthogonality relation, reducing δ_d requires that the colored loading level β_d be increased**
- **In the limit of $\delta_d \to 0$ (true orthogonality of weights to the clutter model), $\beta_d \to \infty$**
- **This can be demonstrated directly by comparing the quadratic constraint weight solution with a multiple linear constraint weight solution that enforces orthogonality explicitly**

Pre-Filter Interpretation
Full-DoF STAP

- **Colored loading beamformer can be expressed as:**

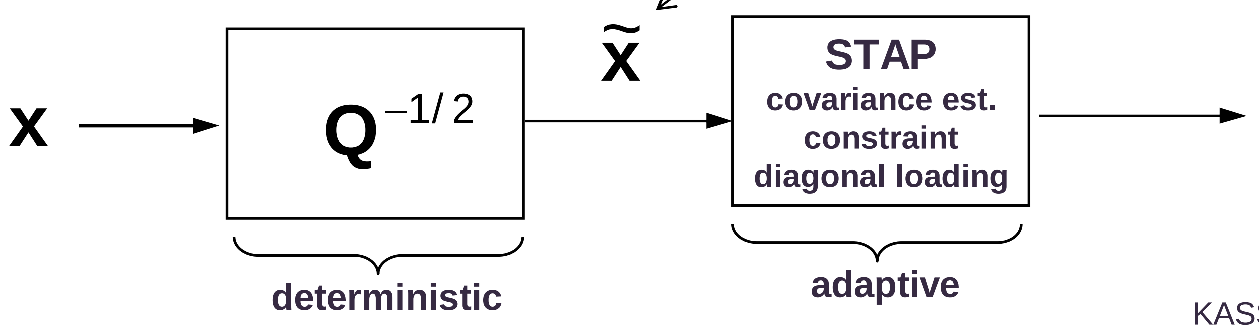
$$\mathbf{w} = \frac{\mathbf{Q}^{-1/2}(\mathbf{Q}^{-1/2}\mathbf{R}_{\mathbf{xx}}\mathbf{Q}^{-1/2} + \mathbf{I})^{-1}\mathbf{Q}^{-1/2}\mathbf{v}}{\mathbf{v}^H\mathbf{Q}^{-1/2}(\mathbf{Q}^{-1/2}\mathbf{R}_{\mathbf{xx}}\mathbf{Q}^{-1/2} + \mathbf{I})^{-1}\mathbf{Q}^{-1/2}\mathbf{v}}$$

- **This filtering solution is equivalent to *deterministic* pre-filtering followed by *adaptive* processing (i.e., 2 stages)**

$$\tilde{\mathbf{x}} = \mathbf{Q}^{-1/2}\mathbf{x} \qquad \tilde{\mathbf{v}} = \mathbf{Q}^{-1/2}\mathbf{v}$$

$$\tilde{\mathbf{w}} = \frac{(\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} + \mathbf{I})^{-1}\tilde{\mathbf{v}}}{\tilde{\mathbf{v}}^H(\mathbf{R}_{\tilde{\mathbf{x}}\tilde{\mathbf{x}}} + \mathbf{I})^{-1}\tilde{\mathbf{v}}}$$

it will generally be easier to estimate the covariance of the pre-filtered data than the original data because it is likely to have a lower *effective* rank



Reduced-DoF STAP

- **For many systems it is not possible to use full-DoF STAP due to limited computational resources and sample support**
- **A common approach is to break the full-DoF problem into a number of smaller reduced-DoF problems via an** NMxD (D < NM**) transformation $\mathbf{H}_m$ on the data:**

$$\mathbf{x}_m = \mathbf{H}_m^H \mathbf{x} \qquad \mathbf{w}_m = \kappa \mathbf{R}_m^{-1} \mathbf{v}_m$$

$$\mathbf{v}_m = \mathbf{H}_m^H \mathbf{v} \qquad y_m = \mathbf{w}_m^H \mathbf{x}_m$$

- **The transformation $\mathbf{H}_m$ can also be applied to the clutter covariance model and thermal noise:**

$$\mathbf{R}_{c,m} = \mathbf{H}_m^H \mathbf{R}_c \mathbf{H}_m \qquad \mathbf{R}_{n,m} = \mathbf{H}_m^H \mathbf{R}_n \mathbf{H}_m$$

thermal noise covariance

- **This presentation will focus on multi-bin element space post-Doppler STAP**

$$\mathbf{R}_n = \mathbf{I}_{NMxNM} \qquad \mathbf{R}_{n,m} = \mathbf{H}_m^H \mathbf{H}_m = \mathbf{I}_{DxD}$$

Knowledge-Aided Quadratic Constraints
Reduced-DoF STAP

- Similar to the full-DoF case we can incorporate the reduced-DoF covariance model as a quadratic constraint

$$\min_{\mathbf{w}_m} \quad E\{|\mathbf{w}_m^H \mathbf{x}_m|^2\} \quad \text{s.t.} \quad \begin{cases} \mathbf{w}_m^H \mathbf{v}_m = 1 \\ \mathbf{w}_m^H \mathbf{R}_{c,m} \mathbf{w}_m \leq \delta_{d,m} \\ \mathbf{w}_m^H \mathbf{w}_m \leq \delta_{L,m} \end{cases}$$

want weights to be "orthogonal" to the reduced-DoF *a priori* clutter model

this is the KA part

- Gives:

$$\mathbf{w}_m = \frac{(\mathbf{R}_m + \beta_{d,m}\mathbf{R}_{c,m} + \beta_{L,m}\mathbf{I})^{-1}\mathbf{v}_m}{\mathbf{v}_m^H(\mathbf{R}_m + \beta_{d,m}\mathbf{R}_{c,m} + \beta_{L,m}\mathbf{I})^{-1}\mathbf{v}_m} = \frac{(\mathbf{R}_m + \mathbf{Q}_m)^{-1}\mathbf{v}_m}{\mathbf{v}_m^H(\mathbf{R}_m + \mathbf{Q}_m)^{-1}\mathbf{v}_m}$$

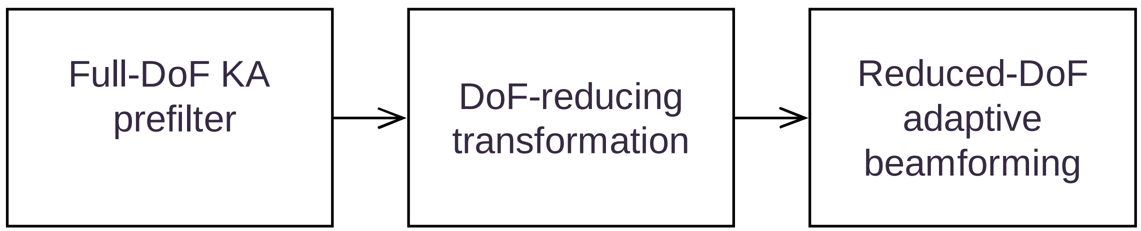
"colored loading"

- Same form as full-DoF case (i.e., colored loading)
- Can also be shown to be a prefilter on the reduced DoF data
- Can be implemented in the data domain

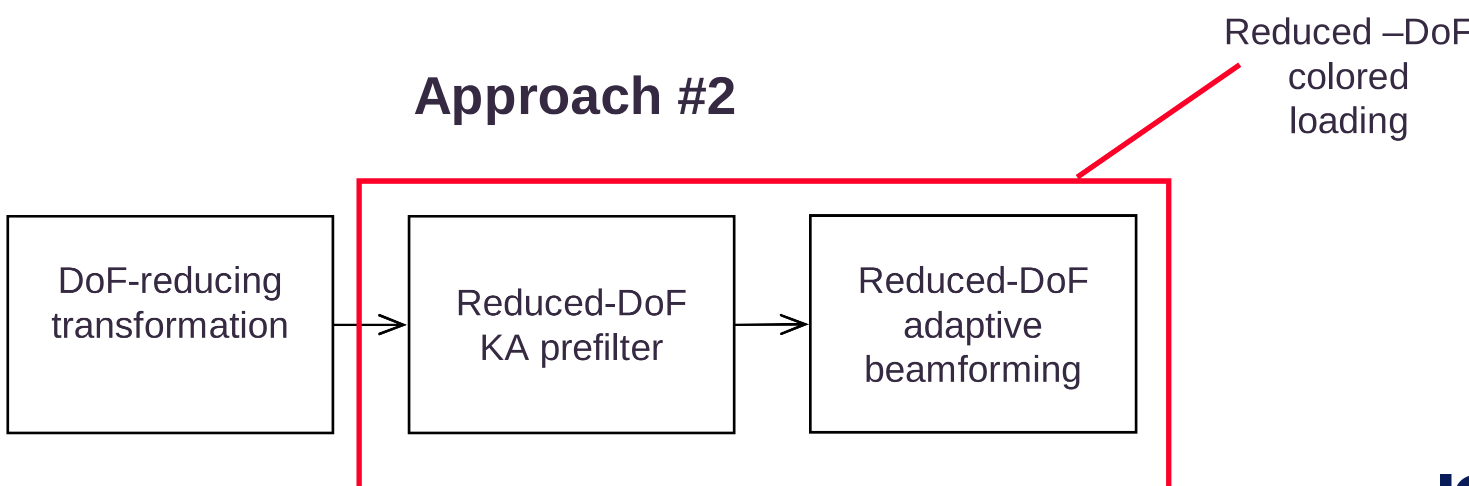
Observation

- **Two approaches to reduced DoF processing with knowledge-aided pre-filters:**

Approach #1



Approach #2



Implementation

- **Typically, the loading matrix Q will be Hermitian and positive-definite so that its Cholesky decomposition exists**

Full DoF
$$\mathbf{Q} = \beta_d \mathbf{R}_c + \beta_L \mathbf{I} = \mathbf{Q}^{1/2} \mathbf{Q}^{1/2} = \mathbf{C}\mathbf{C}^H$$

Reduced DoF
$$\mathbf{Q}_m = \beta_{d,m} \mathbf{R}_{c,m} + \beta_{L,m} \mathbf{I} = \tilde{\mathbf{C}}\tilde{\mathbf{C}}^H$$

$\mathbf{C}$ and $\tilde{\mathbf{C}}$ are Lower Triangular

- **Approach #1, full-DoF pre-filter THEN reduced DoF/beamform**

$$\tilde{\tilde{\mathbf{x}}}_1 = \mathbf{H}_m^H \mathbf{C}^{-1} \mathbf{x} \equiv \mathbf{A}_1 \mathbf{x}$$

- **Approach #2, reduced DoF THEN pre-filter/beamform**

$$\tilde{\tilde{\mathbf{x}}}_2 = \tilde{\mathbf{C}}^{-1} \mathbf{H}_m^H \mathbf{x} \equiv \mathbf{A}_2 \mathbf{x}$$

- **If loading matrix is constant with range, both sets of combined pre-filter/reduced DoF matrices can be pre-computed once**
- **Both can be efficiently implemented in data domain by augmenting data matrix with identity matrix and using QR decomposition**

Implementation, Cont'd

- **However, if Q is a function of range, then pre-filter must be computed several times → Cholesky decomp scales as O(DoF)^3; more expensive for full-DoF**
- **For Approach # 2, pre-filtering can be avoided by equivalently color-loading reduced DoF data**

$$\tilde{\mathbf{x}} = \mathbf{H}_m^H \mathbf{x} \quad \Rightarrow \quad \text{Then color-load with} \quad \tilde{\mathbf{Q}}$$

- **Augment data matrix with Cholesky decomposition of reduced color-loading matrix; then perform efficient QR decomposition**
  - As efficient as diagonal-loading-only in data domain
  - Avoids application of inverse Cholesky matrix
- **If color-load, instead of pre-filter, in data domain for full-DoF, as in Approach # 1, an additional Cholesky decomposition will be required after the reduced DoF transformation since**

$$\tilde{\mathbf{C}} \neq \mathbf{H}_m^H \mathbf{C} \qquad \text{(in general)}$$

- **Appears Approach # 2 may be computationally less expensive than #1; pursue # 2 here; examine performance of # 1 vs # 2 in future**

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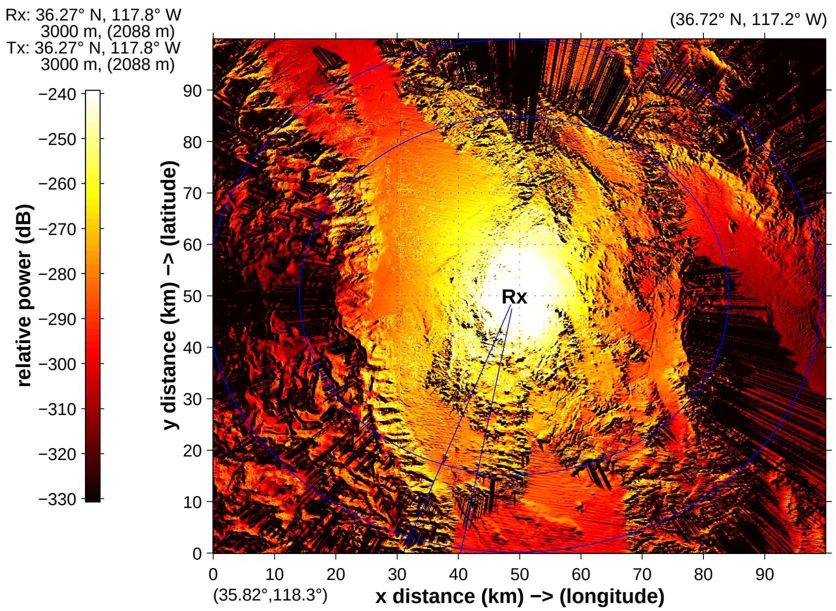
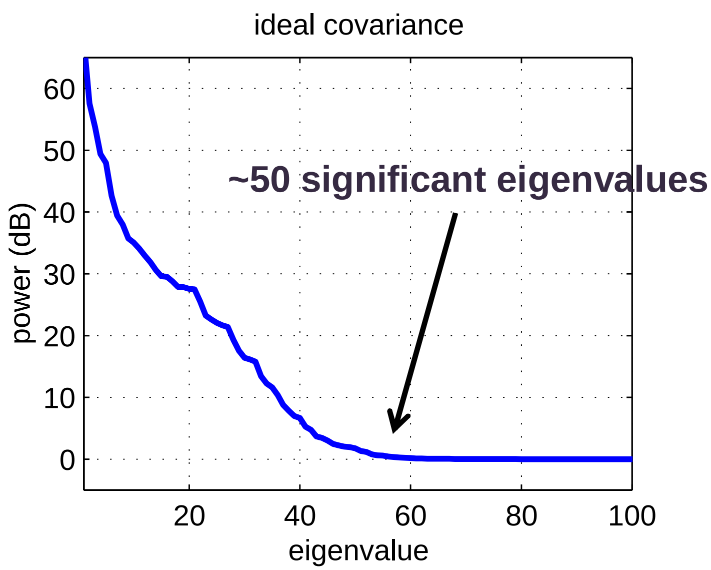
KASSPER Simulated Data Cube



Parmeter	Value
RF frequency	1240 MHz
Bandwidth	10 MHz
PRF	1984 Hz
Peak Power	15 kW
Duty factor	10%
Noise figure	5 dB
System losses	9 dB
Platform speed	100 m/s
Platform altitude	3 km
Transmit aperture	8 vertical x 11 horizontal
Receive aperture*	8 vertical x 1 horizontal
Horizontal antenna spacing	10.9 cm
Vertical antenna spacing	14.07 cm
Number of receive sub-apertures	11
Front-to-back ratio	25 dB



- **Site-specific data set generated under KASSPER program**
- **Heterogeneous clutter, ground vehicles, ICM, calibration errors**
- **We will focus on the problem of detecting slow moving targets in heterogeneous clutter → work with clutter-only data**

Algorithm Details

- **Assume a ring of scatterers every 0.2° around the platform at the desired range bin**
  - No knowledge about: terrain, calibration errors (~5°-10° phase errors), ICM, backlobe level, Tx pattern
  - Only platform heading, speed, and PRF are assumed known
- **Compute a matrix that represents the ground clutter (subspace):**

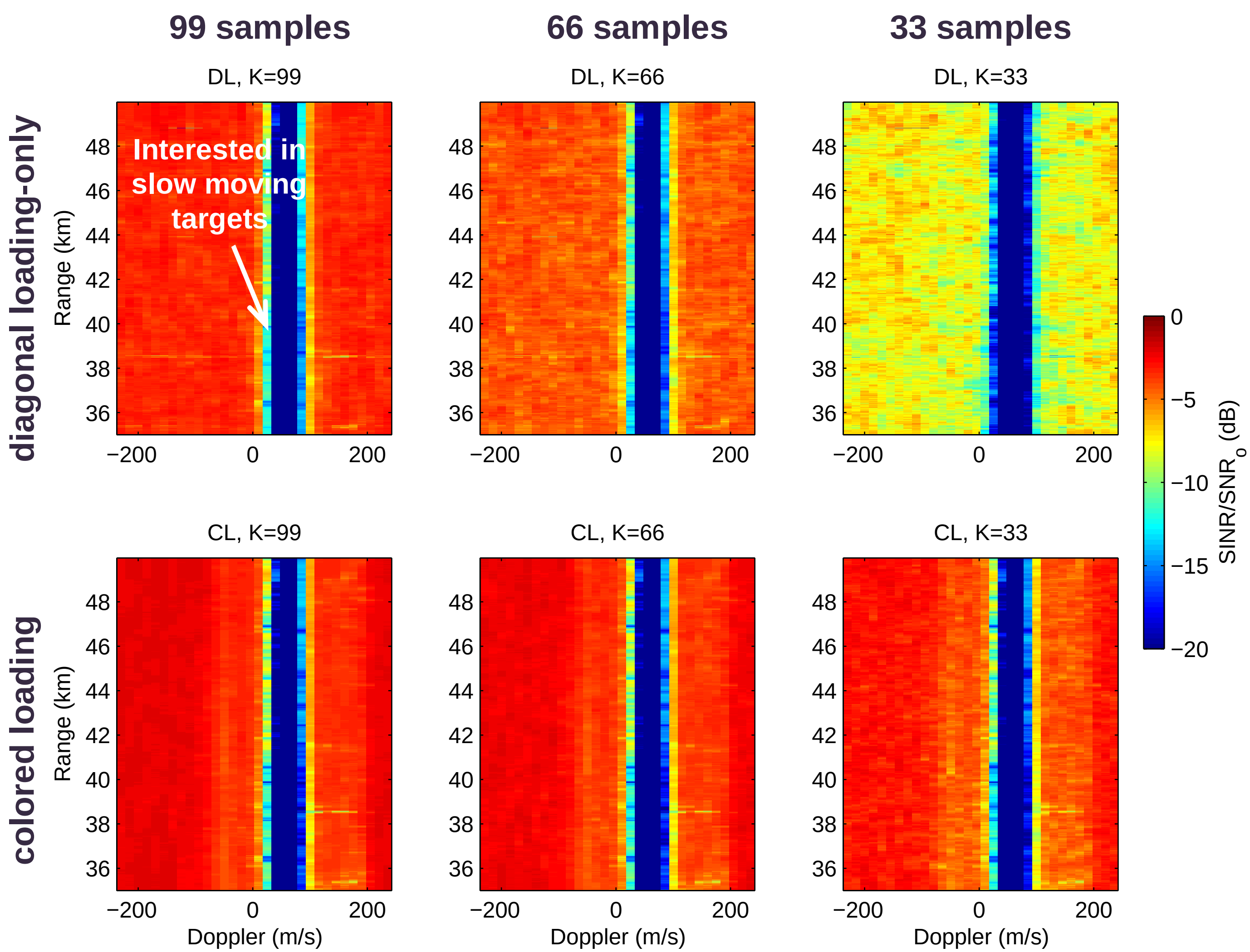
$$\mathbf{R}_c = \sum_{p=1}^{N_c} \mathbf{v}(\theta_p, f_p)\mathbf{v}(\theta_p, f_p)^H \qquad \mathbf{R}_{c,m} = \mathbf{H}_m^H \mathbf{R}_c \mathbf{H}_m$$

- **This form is efficient to compute but not as accurate as the true ideal covariance which will include information about the terrain**
- **Scale this matrix and add to the diagonally-loaded reduced-DoF sample covariance matrix:**

$$\mathbf{w}_m = \kappa\left(\mathbf{R}_{s,m} + \beta_{L,m}\mathbf{I} + \beta_{d,m}\mathbf{R}_{c,m}\right)^{-1}\mathbf{v}_m$$

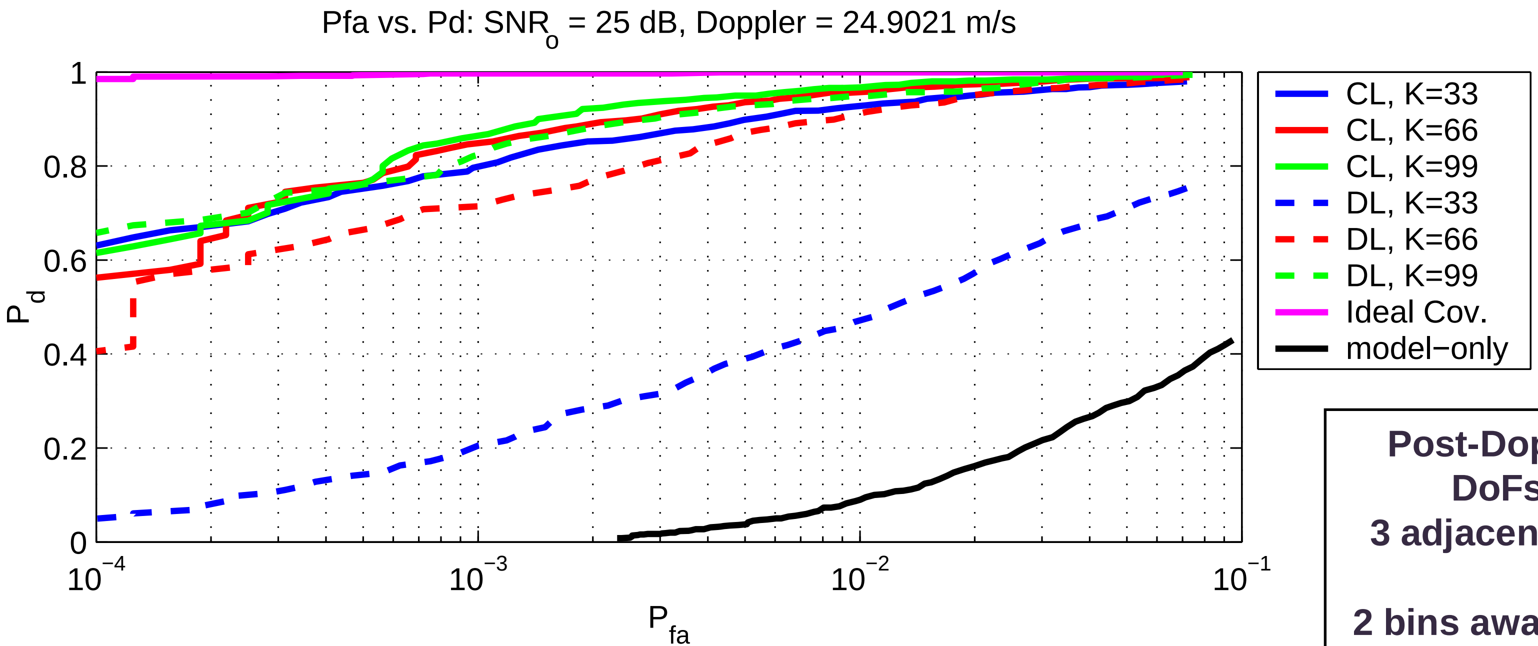
- **Reduced-DoF implementation is multi-bin element space post-Doppler STAP with untapered and orthogonal Doppler filters**

SINR Loss Surfaces



- **Loading levels:**
  β_L = 0 dB
  β_d = 30 dB
- **Colored loading beamformer is more robust to reductions in sample support**
- **Post Doppler element space**
  – 3 bins
  – 11 elements

Detection Performance Summary
("endo-clutter")

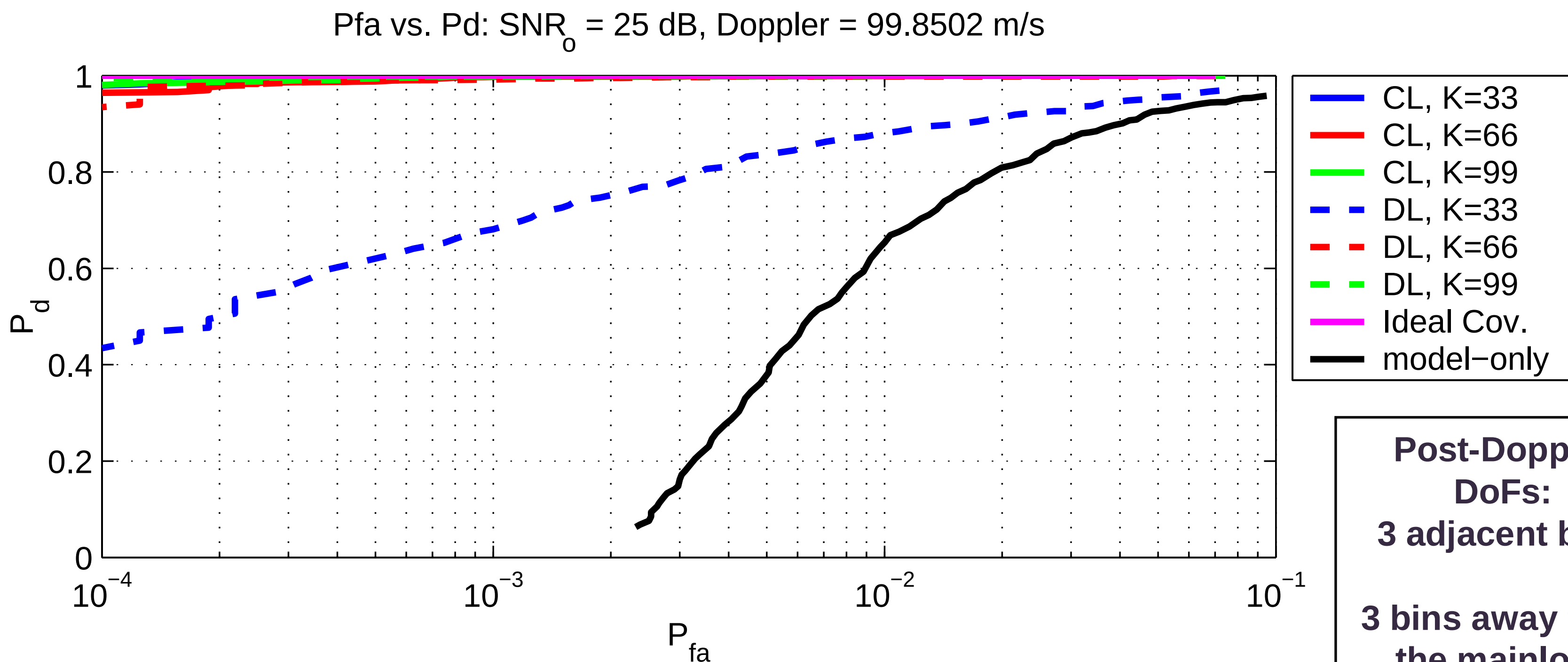


Post-Doppler DoFs:
3 adjacent bins

2 bins away from the mainlobe clutter bin

- **Detector includes median CFAR normalization of the beamformer output prior to thresholding**
- **No targets in the secondary beamformer or CFAR training data**
- **1000 Injected test targets: all ranges, Doppler = 24.90 m/s, Target SNR is 25 dB at closest range bin (~5 dBsm)**
- **Colored loading beamformer is more robust as sample support is reduced**

Detection Performance Summary
(“exo-clutter”)



Post-Doppler DoFs:
3 adjacent bins

3 bins away from the mainlobe clutter bin

- **Same result as previous slide except injected target Doppler is 99.85 m/s**
- **All the beamformers perform well when target is separated from the mainbeam clutter**
- **Use the most computationally efficient algorithm in these Doppler bins**

Summary

- **A method for incorporating *a priori* knowledge in the space-time beamformer solution using quadratic constraints has been presented and extended to reduced-DoF STAP implementations**
- **Quadratic constraint solution results in "colored" loading which can be implemented efficiently in the data domain and offers a "blending" between adaptive and deterministic filtering**
- **The fidelity of the colored loading matrix will depend on the available *a priori* knowledge sources and computational resources**
- **The technique was applied to KASSPER site-specific simulation data and shown to result in more robust performance near the mainbeam clutter → improved MDV performance**